SOLUTIONS: PROBLEM SET 3

ELECTRIC CURRENT and DIRECT CURRENT CIRCUITS

PART A: CONCEPTUAL QUESTIONS

A. If we connect them in series, $R_{eq} = 300\Omega$.
If we connect them in parallel, $R_{eq} = 30\ \Omega$
Therefore, in order to obtain a 150 Ω resistance, we have to connect the resistors in parallel and in series…
Connecting two in parallel: $R_{eq1} = 50\ \Omega$
Connecting R_{eq1} in series with R: $R_{eq} = 150\ \Omega$.

B. c) The wire has essentially zero resistance, compared to the light bulb, and is in parallel with it. Thus, almost all charges flow through the wire, and practically none through the bulb.

C. Circuit 1: f) The potential difference across the branch of the circuit containing the switch is zero. Thus, there is no current through it when the switch is closed, and nothing changes.

Circuit 2: f) Because the light bulbs are identical, the potential difference across each is 12 V, and so nothing happens with the switch is closed.

D. Using the circuit:

a) Dimmer	b) Increase	c) Decrease	d) Increase
e) Increase	f) Decrease	g) Nothing	h) Increase
i) Increase	j) Decrease		

E. The current will be the same because in series, the current in the branch does not change. Even if the light bulbs are different, the current across the resistors will be the same.

F. The current will be different because the light bulbs are in parallel. The current measured between the battery will be equal to the sum of the current in each branch. The current will be greater if the resistor is smaller.

G. In parallel, the current is different but the voltage is the same.

a) The reading on the voltmeter will be the same since the voltage is the same in parallel branches.

b) L_2 since the resistance is less and in order to have the same voltage across the branch the current has to be greater.

c) Note, c) and d) are really the same question. A light bulb which dissipates more energy will be brighter. L_2 will be brighter because it dissipates energy at the rate $P = \frac{V^2}{R}$ and L_2 has less resistance than L_1 (but the same V in parallel).

H. We have to remember the relationship between L, d and R: $R = \rho \frac{L}{A} = \rho \frac{L}{\pi (d/2)^2}$

a) If L is doubled, the resistance R will also double. As long as the diameter is the same. $R \propto L$

b) If d is doubled, the resistance R decreases by a factor of 4. $R \propto d^{-2}$.

c) If L and d are doubled, the resistance R will decrease by a factor of 2.

I. Let's consider a D-cell battery. This is a device that chemically creates a charge differential; one end of the battery is positively charged, and the other is negatively charged, so it has a voltage (typically 1.5V for alkaline cells). The conductor (air) that separates the positive and the electrons prevents the electrons from jumping straight to the positive charge and neutralizing the battery. Air is one of the best insulators around. Now if you attach a wire from one terminal to the other, you have replaced the high resistance of the air with the low resistance of the wire, and you will get a very high current as a result through the wire. It is the chemical reaction between the conductors and the electrolyte, one electrode (the cathode) becomes positively charged and the other, the anode becomes negatively charged.

J. Wrong. When a battery is new it contains stored up chemical energy which can drive charge round a circuit. every atom has charge and the number of atoms in the battery do not change. More to the point, the actual electric current involves a flow of charge, but the battery does not run out of these charges either. There are charge carriers all over the whole circuit, in the wires, the battery, any device connected to the circuit etc and when the switch is closed to turn the current on ALL the charges throughout the whole circuit start moving at once. Over the whole period of use, the number of charges which go out of ONE end of the battery is the same as the number into the OTHER end. You can imagine it like a piece of string stretched round some pulley wheels, when you turn on (pull the string) the string everywhere starts moving at the same moment.

PART B: NUMERICAL QUESTIONS

QUESTION 1

We obtain the resistance R_A of the conductor:

$$R_A = \rho \frac{L}{\pi r_A^2}$$

with r_A radius of conductor A
r_{out} radius of outer conductor B
r_{in} radius of inner conductor B

Therefore, the transversal area is $\pi(r_{out}^2 - r_{in}^2)$

The ratio between A and B will be:

$$\frac{R_A}{R_B} = \frac{r_{out}^2 - r_{in}^2}{r_A^2} = \frac{(1.0mm)^2 - (0.50mm)^2}{(0.50mm)^2} = 3.0$$

QUESTION 2

In order to find the carrier charge density needed in the current density equation, we must find the density of atoms.

N: number of atoms
N_A: avogadro's number (6.02×10^{23} mol^{-1})
m: mass
M: element units

$$\frac{N}{N_A} = \frac{m}{M} \rightarrow N = \frac{N_A m}{M}$$

The mass density of substance is $\rho = m \div Volume \rightarrow Volume = m \div \rho$.
The number density of atoms, $n = N \div Volume = N\rho \div m = (\rho N_A) \div M$

With numerical values: $n = \dfrac{8.9\times10^3 kg/m^3 \cdot 6.02\times10^{23} atoms/mol}{63.5\times10^{-3} kg/mol} = 8.5\times10^{28} atoms/m^3$

The current density is given from $I = qnAv_d$
q: charge
n: number density of charges
A: cross-sectional area
v_d : drift velocity

we solve for v_d: $v_d = \dfrac{I}{qnA} = \dfrac{10A}{e\cdot 8.5\times10^{28} atoms/m^3 \cdot 5\times10^{-6} m^2} = 1.5\times10^{-4} m/s$

QUESTION 3

We first need to find the resistance of each resistors from $P = \frac{V^2}{R}$

We have: $R_1 = \frac{120^2}{60} = 240\Omega$ and $R_2 = \frac{120^2}{90} = 160\Omega$

a) when the resistors are connected in <u>series</u>, the current through them is the same and equal to:

$$I = \frac{\varepsilon}{R_1 + R_2} = \frac{120V}{400\Omega} = 0.3A$$

The voltage of each light will be:

$$V_1 = R_1 I = 240\Omega \cdot 0.3A = 72V$$
$$V_2 = R_2 I = 160\Omega \cdot 0.3A = 48V$$
$$V_{tot} = V_1 + V_2 = 72 + 48 = 120V$$

The power dissipated in each resistor is:

$$P_1 = I^2 R_1 = 21.6W$$
$$P_2 = I^2 R_2 = 14.4W$$
$$P_{tot} = I^2 R_{eq} = P_1 + P_2 = 36.0W$$

b) When the resistors are connected in <u>parallel</u>, the voltage across them is the same and equal to:

$$V_{tot} = 120V = V_1 = V_2$$

The equivalent resistance is found from : $R_{eq} = (R_1^{-1} + R_2^{-1})^{-1} = 96\Omega$

The current in the circuit is $I_{tot} = \frac{\varepsilon}{R_{eq}} = \frac{120V}{96\Omega} = 1.25A$

The current in each light bulb is:

$$I_1 = \frac{V}{R_1} = \frac{120V}{240\Omega} = 0.5A$$
$$I_2 = \frac{V}{R_2} = \frac{120V}{160\Omega} = 0.75A$$
$$I_{tot} = I_1 + I_2 = 1.25A$$

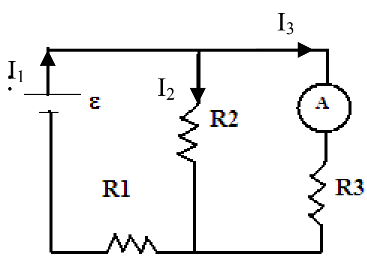
The power dissipated in each resistor is:

$$P_1 = I_1^2 R_1 = 60W$$
$$P_2 = I_2^2 R_2 = 90W$$
$$P_{tot} = I_{tot} R_{eq} = P_1 + P_2 = 150W$$

c) Since the rate of energy dissipated by the circuit is greater when the light bulbs are in parallel ($P_{parallel} > P_{series}$), we conclude that the brightness will also be greater in parallel.

d) The light bulbs in series are more economical because the power dissipated in less. However, there are some inconvenient to this. If one light bulb burns or is unscrewed in series, not more current flows!

QUESTION 4



a) The current in R_1:

$$I_1 = \frac{\varepsilon}{R_1 + (R_2^{-1} + R_3^{-1})^{-1}} = \frac{5.0V}{2.0\Omega + (4.0^{-1} + 6.0^{-1})^{-1}} = 1.14A$$

therefore, the current in R_3 is:

$$I_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - I_1 R_1}{R_3} = \frac{5.0V - (1.14A \cdot 2.0\Omega)}{6.0\Omega} = 0.45A$$

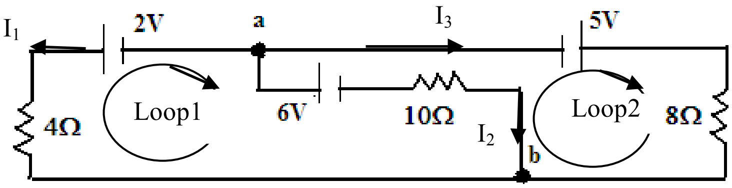
b) We exchange R_1 and R_3 in the above equation:

$$I_3 = \frac{\varepsilon}{R_3 + (R_1^{-1} + R_2^{-1})^{-1}} = \frac{5.0V}{6.0\Omega + (2.0^{-1} + 4.0^{-1})^{-1}} = 0.68A$$

and

$$I_1 = \frac{\varepsilon - V_3}{R_1} = \frac{\varepsilon - I_3 R_3}{R_1} = \frac{5.0V - (0.68A \cdot 6.0\Omega)}{2.0\Omega} = 0.45A$$

QUESTION 5



We have three unknowns (I_1, I_2 and I_3) so three equations are needed.

a) With Kichhoff's law of current and from the above currents in the circuit:

$$(1)\ I_1 + I_2 + I_3 = 0$$

But, the direction of one current is incorrect… It will not cause any difference because the current will be negative. Let apply the voltage law on loop 1:

$$(2)\qquad \Sigma V = -2.0V - 6.0V - 10.0\Omega \cdot I_2 + 4.0\Omega \cdot I_1 = 0$$

The emf are negative since we encounter the + side first. From (2):

$$(3)\qquad I_2 = \frac{-8.0V + 4.0\Omega \cdot I_1}{10.0\Omega} = -0.8A + 0.4I_1$$

from loop 2

$$(4)\qquad \Sigma V = 6.0V - 8.0\Omega \cdot I_3 + 10.0\Omega \cdot I_2 + 5.00 = 0$$

we substitute I_3 from equation (1): $I_3 = -I_1 - I_2$ in equation (4) and solve for I_2:

$$(5)\qquad 11V - 8.0\Omega \cdot (-I_1 - I_2) + 10.0\Omega \cdot I_2 = 0$$

$$(6)\qquad I_2 = \frac{-11.0V}{18.0\Omega} - \frac{8.0\Omega}{18.0\Omega} I_1 = -0.611A - 0.444 \cdot I_1$$

Since equation (3) equals equation (6)

$$-0.8A + 0.4 \cdot I_1 = -0.611A - 0.444 \cdot I_1$$

$$I_1 = 0.224\text{A}$$

Therefore we can substitute in equation (6) and (1)

$I_2 = -0.8A + 0.4 \cdot 0.224A = -0.710A$ in the figure, I_2 has to be on the other direction

$I_3 = -I_1 - I_2 = -0.224A - (-0.710A) = 0.486A$

Verify the results:

From the external loop:
$\Sigma V = -2.0V + 5.0V - 8.0\Omega \cdot I_3 + 4.0\Omega \cdot I_1 = 0$

$3V - 8.0\Omega \cdot 0.486A + 4.0\Omega \cdot 0.224A = 0$

b) The potential difference between points a and b is:

$$V_b - 10.0\Omega \cdot I_2 + 6.0V = V_a$$
$$V_b - V_a = 10.0\Omega \cdot 0.710 - 6.0V = 1.1V$$

QUESTION 6

a) The current when the resistors are in series will be:

$$I = \frac{\varepsilon}{R_1 + R_2 + R_3} = \frac{10.0V}{100\Omega + 220\Omega + 680\Omega} = 10mA$$

The current will be the same in all resistors in series. However, the voltmeter will have a different reading, depending on the resistance.

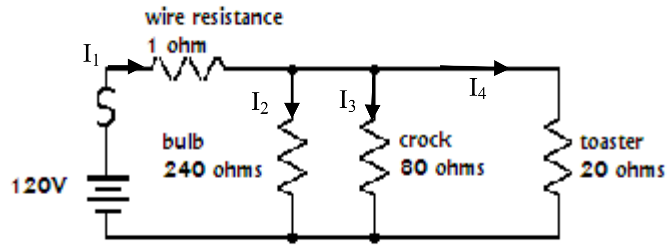
$$V_1 = IR_1 = 1.0V$$
$$V_2 = IR_{23} = 9.0V$$
$$V_3 = IR_3 = 6.8V$$

b) Voltmeter 2, is connected across resistors 2 and 3.

Since $V_2 = I(R_2 + R_3)$ and $I = \frac{\varepsilon}{R_1 + R_2 + R_3}$

$$V_2 = \frac{\varepsilon}{R_1 + R_2 + R_3} \cdot (R_2 + R_3) = \frac{\varepsilon(R_2 + R_3)}{(R_1 + R_2 + R_3)}$$

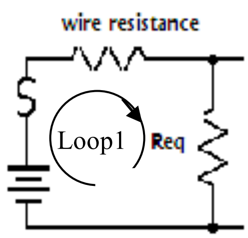
QUESTION 7



The light bulb, the toaster and the crock are in parallel; we can find the equivalent resistance:

$$R_{eq} = (R_{bulb}^{-1} + R_{crock}^{-1} + R_{toaster}^{-1})^{-1} = (240^{-1} + 80^{-1} + 20^{-1})^{-1} = 15\Omega$$

Let consider the left hand side of the circuit has loop 1.



We have a single loop:

$$\Sigma V = 120V - R_{wire} I_1 - R_{eq} I_1 = 120V - 1\Omega \cdot I_1 - 15\Omega \cdot I_1 = 0$$

$$I_1 = 7.5A$$

The current law: $I_2 + I_3 + I_4 = I_1$ and because the branches are in parallel, the voltage across each resistor is the same.

$$V_2 = V_3 = V_4 = V_{eq} = 15\Omega \cdot I_1 = 112.5V$$

$$I_2 = \frac{112.5V}{240\Omega} = 0.46A$$

$$I_3 = \frac{112.5V}{80\Omega} = 1.4A$$

$$I_4 = \frac{112.5V}{20\Omega} = 5.6A$$

Verify the results:

$$I_1 = I_2 + I_3 + I_4 = 7.5A = 0.46A + 1.4A + 5.6A = 7.5A$$

QUESTION 8

a) before the switch is closed, resistors R_1 and R_2 are in series and R_3 and R_4 are also in series. The equivalent resistance for each combination is:

$$R_{12} = R_1 + R_2 = 4.00\Omega + 10.0\Omega = 14.0\Omega$$
$$R_{34} = R_3 + R_4 = 12.0\Omega + 10.0\Omega = 22.0\Omega$$

R_{12} and R_{34} are in parallel, therefore the equivalent resistance of the system is:

$$(R_{34}^{-1} + R_{12}^{-1})^{-1} = R_{eq} = 8.55\Omega$$

From Ohm's law: $V = RI \rightarrow I = \dfrac{V}{R_{eq}} = \dfrac{25.0V}{8.55\Omega} = 2.92A$

The current in each branch will be:

$$I_{12} = \frac{V_{12}}{R_{12}} = \frac{25.0V}{14.0\Omega} = 1.78A$$
$$I_{34} = \frac{V_{34}}{R_{34}} = \frac{25.0V}{22.0\Omega} = 1.14A$$

(and $I_1 = I_2$ and $I_3 = I_4$)

Check the result:

$$I_{tot} = I_{12} + I_{34} = 2.92A$$

b) The switch is now closed. We have to find the equivalent resistance of the circuit where R_1 and R_3 are in parallel and R_2 and R_4 are in parallel.

$$R_{13} = (R_1^{-1} + R_3^{-1})^{-1} = (4.00^{-1} + 12.0^{-1})^{-1} = 3.00\Omega$$
$$R_{24} = (R_2^{-1} + R_4^{-1})^{-1} = (10.0^{-1} + 10.0^{-1})^{-1} = 5.00\Omega$$

R_{13} and R_{24} are in series. The equivalent resistance of the circuit is

$$R_{eq} = R_{13} + R_{24} = 3.00 + 5.00 = 8.00\Omega$$

From Ohm's law: $V = RI \rightarrow I = \dfrac{V}{R_{eq}} = \dfrac{25.0V}{8.00\Omega} = 3.13A$

The current through each resistor; we have to find the voltage across each resistor, and the voltage across R_1 and R_3 is the same, the voltage across R_2 and $R_4$is the same.

$$I_1 = \frac{V_{13}}{R_1} = \frac{IR_{13}}{R_1} = \frac{3.13A \cdot 3.00\Omega}{4.00\Omega} = 2.34A$$
$$I_2 = \frac{V_{24}}{R_2} = \frac{IR_{24}}{R_2} = \frac{3.13A \cdot 5.00\Omega}{10.0\Omega} = 1.56A$$
$$I_3 = \frac{V_{13}}{R_3} = \frac{IR_{13}}{R_3} = \frac{3.13A \cdot 3.00\Omega}{12.0\Omega} = 0.78A$$
$$I_4 = \frac{V_{24}}{R_4} = \frac{IR_{24}}{R_4} = \frac{3.13A \cdot 5.00\Omega}{10.0\Omega} = 1.56A$$

Check the result:

$$I = I_1 + I_3 = I_2 + I_4$$
$$3.13A = 2.34A + 0.78A = 1.56A + 1.56A$$
$$3.13 \approx 3.12A \approx 3.12A$$

c) Before the switch is closed: R_1 and R_2 are in series and R_3 and R_4 are in series. From the results in part a), we can find the voltage across each resistor.

$$V_1 = R_1 I_1 = I_{12} R_1 = 1.78A \cdot 4.00\Omega = 7.14V$$
$$V_2 = R_2 I_2 = I_{12} R_2 = 1.78A \cdot 10.0\Omega = 17.86V$$
$$V_3 = R_3 I_3 = I_{34} R_3 = 1.14A \cdot 12.0\Omega = 13.64V$$
$$V_4 = R_4 I_4 = I_{34} R_4 = 1.14A \cdot 10.0\Omega = 11.36V$$

Check the result:

$$V_{tot} = V_1 + V_2 = V_3 + V_4$$
$$25.0V = 7.14V + 17.86V = 13.64V + 11.36V$$
$$25.0V = 25.0V = 25.0V$$

The voltage at points a and b:

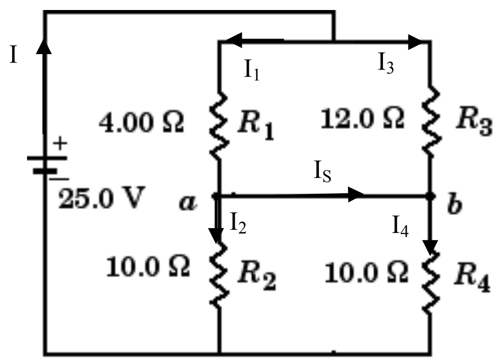
$$V_a = 25.0V - V_1 = 25.0V - 7.14V = 17.9V$$
$$V_b = 25.0V - V_3 = 25.0V - 13.64V = 11.4V$$

The potential difference between point a and b:

$$V_{ab} = |V_b - V_a| = |11.36V - 17.85V| = 6.49V$$

After the switch is closed, $V_{ab} = 0$ because connecting points a and b forces the potential at points and b to be the same, so that their potential difference is zero.

d)



I_S is the current through the switch

$$I_1 = I_S + I_2$$

$$I_S = I_1 - I_2 = 2.34A - 1.56A = 0.78A$$

As a double check (!!) we apply Kirchhoff's current rule at point b

$$I_S + I_3 = I_4$$

$$I_S = I_4 - I_3 = 1.56A - 0.78A = 0.78A$$

The current is flowing from a to b (because the current found is positive)

QUESTION 9

The total power when the if the electric frying pan, the coffee and the toaster are used:

$$P_{tot} = P_{pan} + P_{coffee} + P_{toaster} = 1000W + 600W + 700W = 2300W$$

The current flowing in the circuit is:

$$P = VI \rightarrow I = P \div V = 2300 \div 120 = 19.1A$$

$I_{system} \langle I_{fuse}$ therefore, Jim should make the toasts (Jenny will be happy!)

However, if the overhead light is on, the total power will increase;

$$P_{tot} = P_{pan} + P_{coffee} + P_{toaster} + P_{light} = 1000W + 600W + 700W + 100W = 2400W$$

The current flowing in the circuit is:

$$P = VI \rightarrow I = P \div V = 2400 \div 120 = 20A$$

$I_{system} = I_{fuse}$ therefore, the overhead light should not be on.

QUESTION 10

As one of the voltmeters indicates the voltage exceeding the battery *emf*, the *black box* contains at least one battery. Let us try the simplest combination of one battery and one resistor inside the *black box*.

There are two different voltages across two identical ammeters. The voltage across the left ammeter (10mA) is equal to

$$3.6V - 3.3V = 0.3V$$

It permits us to calculate the voltage across the right ammeter (12mA) which is equal to

$$0.3V \times \frac{12}{10} = 0.36V$$

Now it is possible to calculate the *emf* inside the *black box*:

$$\varepsilon = 3.6 + 0.36V = 3.96V$$

We can compare the currents in two identical voltmeters. The current of 12mA splits in the node N into two currents: 10 mA through the left ammeter and 2 mA through the left voltmeter. The internal resistance of the voltmeter is

$$R = \frac{V}{I} = \frac{3.6}{0.002} = 1.8k\Omega$$

Current in the right voltmeter is:

$$I = \frac{V}{R} = \frac{3V}{1800\Omega} = 1.7mA$$

So, in the loop with the battery in the *black box*, unknown resistor and right voltmeter the following is correct:

$$R_x = \frac{3.96V - 3V}{1.7 \times 10^{-3} A} = 565\Omega$$

The direction of the inbox *emf* must be as it is shown in the following figure: