

201-103-RE - Supplement D
Derivatives of Trigonometric, Exponential and Logarithmic Functions

For each of the following function, find y' :

(1) $y = 3x^2 \sin(8 + 2x)$

(2) $y = (5x - 2) \cos(5x)$

(3) $y = 6x - \frac{5x}{\sin(4x)}$

(4) $y = x^2 \sin(2x) - 4x$

(5) $y = \frac{3x - 1}{\sin(3x)}$

(6) $y = (2 + \sin(2x))(\sec(2x) + 4)$

(7) $y = \frac{\sin(x) + x^2}{4x - \cos(x)}$

(8) $y = 12x^2 + \cos(3 - x)$

(9) $y = \sin \left[(3x - x^2)^2 \right]$

(10) $y = \sqrt[3]{\sin(3x) + \cos(3x) + 2}$

(11) $\cos(3x - y) + 4y + 2x = 15$

(12) $y \cos(x) + x^2 \cos(y) = \pi^2$

(13) $4 \cos(y) + 3x = 6$

(14) $\cos\left(\frac{y}{x}\right) + x^2 = 4$

(15) $\sec(xy^2) - y + 6x = 6$

(16) $\sin(3y - x) + 2 \cos(3y - 1) = 3$

(17) $\tan(4x - y) + 2y = 8$

(18) $\frac{3 \cos(x) - 1}{1 + \cos(y)} = 2 + 3x$

(19) $y = \sin^2(6 - 2x) + x^3$

(20) $y = \sin\left(\frac{x+1}{2x}\right)$

(21) $y = 4^{2x} \sin(3x)$

(22) $y = 7^{4 \sin(x) + x^2}$

(23) $y = \tan(3^{3x} - 1)$

(24) $y = 2^{x+3} \sin(\pi x)$

(25) $y = \frac{\sin(4x)}{e^{3x}}$

(26) $y = 13^{\sin(2x-6)}$

(27) $3 \tan(y) - e^{2x} + 1 = 0$

(28) $4 \tan(3y) - e^{4x} + 1 = 0$

(29) $y = \log_4(2x + \cos(x))$

(30) $y = \sin(\log_5(x) + \pi x)$

(31) $y = \frac{\cos(x)}{\log_{13}(2x+1) + 3}$

(32) $y = \log_2(4 \sin(x) + e^{3x})$

(33) $y = (4x + 3)^{\tan(3-x)}$

(34) $\log_3(2 \tan(x) + 1) = \sin(3y)$

(35) $y = (4 - 3x) \cot(3x)$

(36) $y = 5x - \frac{4x}{\sec(2x)}$

(37) $y = 3x \cos(2x) - 2x^2$

(38) $y = \frac{5x - 3}{\cos(4x)}$

(39) $y = (3 - \cos(3x))(\tan(3x) + 6)$

(40) $y = \frac{3x - \sin(x)}{x^2 + \cos(x)}$

(41) $y = 5x^2 - \sin(1 - x)$

(42) $y = \cos \left[(x^3 - 4x)^2 \right]$

(43) $y = \sqrt[4]{2 \cos(2x) - \sin(2x) - 1}$

(44) $\sin(2y - x) + 3y^2 = x + 1$

(45) $y^2 \cos(y) + x \cos(y) + \pi^2 = 0$

(46) $6 \sin(y) - 2x + 2 = 0$

(47) $\sin(xy) + y^2 = 4$

(48) $\csc(3y) + \cos(y) - x^2 + 2 = 0$

(49) $\sec(2y - x) + 3y = 3x^2 + 1$

(50) $\frac{\sin(x) + 3}{3 \cos(y) + 1} - 4y = \cos(x) + 1$

(51) $y = \cos^2(15 - 3x) - 3x^2$

(52) $y = \tan\left(\frac{3x}{x-2}\right)$

(53) $y = e^{-x} \cos(2x)$

(54) $y = e^{\sin(x)+x}$

(55) $y = \sin(1 - e^{2x})$

(56) $y = e^{2x+4} \tan(\pi x)$

(57) $y = \frac{\tan(2x)}{e^{3x}}$

(58) $y = e^{-\tan(2-x)}$

(59) $4 \tan(y) + e^{4x} = 1$

(60) $5 \sin(x) - e^{3y} + 1 = 0$

(61) $y = \ln(3x + \sec(x))$

(62) $y = \tan(\pi x^2 - \ln(x))$

(63) $y = \frac{\sin(x)}{4 - \ln(3x + 1)}$

(64) $y = \ln(3 \tan(x) + e^{5x})$

(65) $\ln(3 \sin(x) + 1) = \tan(2y)$

(66) $y = \ln \left[\frac{(x^3 + 1)^2 (\tan(x) + 2)^3}{\sqrt{\cos(x) + 2}} \right]$

$$(67) y = \ln \left[\frac{\sqrt[3]{2x - \cos(x)}}{(\sin(x) + 4)^3 \sqrt{x + 1}} \right] \quad (68) y = \ln \left[\frac{\cos^2(x^2 - 1)}{\sqrt{x + 3}(x^2 + 1)^3} \right] \quad (69) \sin\left(\frac{x}{y}\right) + \cos(xy) + 2y^3 = 17$$

$$(70) \sin(\ln(y)) - \tan(xy) + 3x = 0 \quad (71) e^{\sin(y)} - \cos\left(\frac{y}{x}\right) + 4x = 3 \quad (72) e^{y - \sin(x)} + \frac{\cos(y)}{4x + 1} + 3y = 2$$

Find the higher-order derivatives:

$$(73) \text{ Find } \frac{d^4 y}{dx^4} \text{ if } y = \cos(9 - 3x) \quad (74) \text{ Find } y''' \text{ if } y = \cos\left(\frac{x}{3}\right) \quad (75) \text{ Find } \frac{d^2 y}{dx^2} \text{ if } y = \ln(\cos(3x))$$

$$(76) \text{ Find } y'' \text{ if } y = \ln(\sin^3(x)) \quad (77) \text{ Find } y'' \text{ if } y = e^{\cos(2x) - 1} \quad (78) \text{ Find } y'' \text{ if } y = 3x \sin(2x)$$

$$(79) \text{ Find } \frac{d^2 y}{dx^2} \text{ if } y = \sin(2 \ln(x)) \quad (80) \text{ Find } y'' \text{ if } y = (2x - 1) \cos(3x) \quad (81) \text{ Find } y'' \text{ if } y = \frac{\sin(x) + 3}{\sin(x) + 4}$$

$$(82) \text{ Find } \frac{d^2 y}{dx^2} \text{ if } y = \ln(\sin(5x)) \quad (83) \text{ Find } \left. \frac{d^2 y}{dx^2} \right|_{x=0} \text{ if } y = \ln(\cos^2(2x)) \quad (84) \text{ Find } y''' \text{ if } y = \sin\left(\frac{x}{2}\right)$$

For the following functions, find y' using logarithmic differentiation.

$$(85) y = (x + 1)^{2 \cos(x)} \quad (86) y = (4 + x)^{\sin(4 - x)} \quad (87) y = \frac{(6x + 1)^2 \sqrt[4]{2x^2 + 1}}{e^{1 - \cos(x)}}$$

$$(88) y = \frac{\sin(4x) e^{3 \sin(x)}}{\sqrt[3]{9x + 1}} \quad (89) y = \frac{\sqrt{\sin(3x) + 1}}{\cos^2(x) \sqrt[3]{x^2 + 1}} \quad (90) y = \frac{(x + 2) \sqrt{\cos^3(x)}}{(3x + \cos(2x))^4}$$

$$(91) y = \frac{(\sin(3x) - \cos(2x))^4}{2 \sec(x) (\tan(x) + 2)^2} \quad (92) y = (\sin(3x))^{\frac{1}{x+1}} \quad (93) y = (x^2 + 2)^{\tan(x)}$$

$$(94) y = (\tan(2x) + 3)^{\cos(x)} \quad (95) y = (\sin(3x) + \cos(x))^{\sqrt{x+1}} \quad (96) y = \left(\ln(\cos(x)) + 4 \right)^{\tan(2x)}$$

$$(97) y = \frac{\sqrt[3]{3x + 1} e^{\sin(2x)}}{(x^3 + 1)^3} \quad (98) y = \frac{\cos(2x) \sqrt{4x + 1}}{e^{\sin(3x)}} \quad (99) y = (\tan(x))^{2x}$$

Find the higher order derivatives:

$$(100) \text{ Given } f(x) = \sin(x), \text{ find } f^{(81)}(x)$$

$$(101) \text{ Given } f(x) = \cos(2x - 5), \text{ find } f^{(54)}(x)$$

$$(102) \text{ Given } f(x) = \sin(3x + 1), \text{ find } f^{(75)}(x)$$

$$(103) \text{ Given } f(x) = \cos(3 - 2x), \text{ find } f^{(124)}(x)$$

$$(104) \text{ Given } f(x) = \sin(1 - 6x), \text{ find } f^{(45)}(x)$$

$$(105) \text{ Given } f(x) = x \sin(x), \text{ find } f^{(51)}(x)$$

$$(106) \text{ Given } f(x) = x^{10} + 7x^7 - 3x^3 + 5, \text{ find } f^{(10)}(x)$$

$$(107) \text{ Given } f(x) = 2x^{23} + 17x^{15} - 6x^{11}, \text{ find } f^{(30)}(x)$$

$$(108) \text{ Given } f(x) = \frac{-2}{x^3}, \text{ find } f^{(51)}(x)$$

$$(109) \text{ Given } f(x) = e^{-5x} + x^2, \text{ find } f^{(101)}(x)$$

For each function below, find the x -coordinate(s), if any, for the points at which the graph of f has a horizontal tangent.

$$(110) f(x) = x^2 e^{6x}$$

$$(111) f(x) = 3x e^{1 - 8x^2}$$

$$(112) f(x) = e^x (x - 6)^5$$

$$(113) f(x) = e^{x^2} (2x + 6)^4$$

$$(114) f(x) = e^x \sqrt{2x + 9}$$

ANSWERS:

- (1) $6x \sin(8 + 2x) + 6x^2 \cos(8 + 2x)$ (2) $5 \cos(5x) - (25x - 10) \sin(5x)$ (3) $6 - \frac{5 \sin(4x) - 20x \cos(4x)}{\sin^2(4x)}$
- (4) $2x \sin(2x) + 2x^2 \cos(2x) - 4$ (5) $\frac{3 \sin(3x) - (9x - 3) \cos(3x)}{\sin^2(3x)}$
- (6) $2 \cos(2x)[\sec(2x) + 4] + 2 \sec(2x) \tan(2x)[2 + \sin(2x)]$ (7) $\frac{2x \cos(x) - (x^2 + 4) \sin(x) + 4x^2 - 1}{(4x - \cos(x))^2}$
- (8) $24x + \sin(3 - x)$ (9) $(4x^3 - 18x^2 + 18x) \cos[(3x - x^2)^2]$ (10) $\frac{3 \cos(3x) - 3 \sin(3x)}{3(\sin(3x) + \cos(3x) + 2)^{\frac{2}{3}}}$
- (11) $\frac{3 \sin(3x - y) - 2}{\sin(3x - y) + 4}$ (12) $\frac{y \sin(x) - 2x \cos(y)}{\cos(x) - x^2 \sin(y)}$ (13) $\frac{3}{4 \sin(y)}$ (14) $\frac{2x^3 + y \sin(\frac{y}{x})}{x \sin(\frac{y}{x})}$ (15) $\frac{6 + y^2 \sec(xy^2) \tan(xy^2)}{1 - 2xy \sec(xy^2) \tan(xy^2)}$
- (16) $\frac{\cos(3y - x)}{3 \cos(3y - x) - 6 \sin(3y - 1)}$ (17) $\frac{4 \sec^2(4x - y)}{\sec^2(4x - y) - 2}$ (18) $\frac{3 \sin(x) + 3 \cos(y) + 3}{2 \sin(y) + 3x \sin(y)}$
- (19) $3x^2 - 4 \sin(6 - 2x) \cos(6 - 2x)$ (20) $\cos\left(\frac{x+1}{2x}\right) \frac{-1}{2x^2}$ (21) $4^{2x}(2 \ln(4) \sin(3x) + 3 \cos(3x))$
- (22) $7^{4 \sin(x) + x^2} \ln(7)(2x + 4 \cos(x))$ (23) $3 \ln(3) 3^{3x} \sec^2(3^{3x} - 1)$ (24) $2^{x+3}(\ln(2) \sin(\pi x) + \pi \cos(\pi x))$
- (25) $\frac{4 \cos(4x) - 3 \sin(4x)}{e^{3x}}$ (26) $2 \ln(13) \cos(2x - 6) 13^{\sin(2x-6)}$ (27) $\frac{2e^{2x}}{3 \sec^2(y)}$ (28) $\frac{e^{4x}}{3 \sec^2(3y)}$
- (29) $\frac{2 - \sin(x)}{(2x + \cos(x)) \ln(4)}$ (30) $\cos(\log_5(x) + \pi x) \left(\frac{1}{x \ln(5)} + \pi\right)$ (31) $\frac{\frac{-2 \cos(x)}{(2x+1) \ln(13)} - \sin(x) (\log_{13}(2x+1) + 3)}{(\log_{13}(2x+1) + 3)^2}$
- (32) $\frac{4 \cos(x) + 3e^{3x}}{(4 \sin(x) + e^{3x}) \ln(2)}$ (33) $(4x + 3)^{\tan(3-x)} \left[\frac{4 \tan(3-x)}{4x+3} - \sec^2(3-x) \ln(4x+3)\right]$ (34) $\frac{2 \sec^2(x)}{3 \ln(3) \cos(3y)(2 \tan(x) + 1)}$
- (35) $-3 \cot(3x) - 3(4 - 3x) \csc^2(3x)$ (36) $5 - \frac{4 - 8x \tan(2x)}{\sec(2x)}$ (37) $3 \cos(2x) - 6x \sin(2x) - 4x$
- (38) $\frac{5 \cos(4x) + 4(5x - 3) \sin(4x)}{\cos^2(4x)}$ (39) $9 \sec^2(3x) - 3 \cos(3x) + 18 \sin(3x)$
- (40) $\frac{(3 - \cos(x))(x^2 + \cos(x)) - (3x - \sin(x))(2x - \sin(x))}{(x^2 + \cos(x))^2}$
- (41) $10x + \cos(1 - x)$ (42) $-2(x^3 - 4x)(3x^2 - 4) \sin[(x^3 - 4x)^2]$ (43) $\frac{-4 \sin(2x) - 2 \cos(2x)}{4(2 \cos(2x) - \sin(2x) - 1)^{\frac{3}{4}}}$
- (44) $\frac{\cos(2y - x) + 1}{2 \cos(2y - x) + 6y}$ (45) $\frac{-\cos(y)}{2y \cos(y) - y^2 \sin(y) - x \sin(y)}$ (46) $\frac{2}{6 \cos(y)}$ (47) $\frac{-y \cos(xy)}{x \cos(xy) + 2y}$
- (48) $\frac{-2x}{3 \csc(3y) \cot(3y) + \sin(y)}$ (49) $\frac{6x + \sec(2y - x) \tan(2y - x)}{2 \sec(2y - x) \tan(2y - x) + 3}$ (50) $\frac{3 \sin(x) \cos(y) + \sin(x) + \cos(x)}{4 + 12 \cos(y) - 12y \sin(y) - 3 \sin(y) - 3 \cos(x) \sin(y)}$
- (51) $6 \cos(15 - 3x) \sin(15 - 3x) - 6x$ (52) $\sec^2\left(\frac{3x}{x-2}\right) \frac{-6}{(x-2)^2}$ (53) $-e^{-x}(\cos(2x) + 2 \sin(2x))$
- (54) $e^{\sin(x)+x}(\cos(x) + 1)$ (55) $-2e^{2x}(\cos(1 - e^{2x}))$ (56) $e^{2x+4}(2 \tan(\pi x) + \pi \sec^2(\pi x))$ (57) $\frac{2 \sec^2(2x) - 3 \tan(2x)}{e^{3x}}$
- (58) $\sec^2(2 - x)e^{-\tan(2-x)}$ (59) $\frac{-e^{4x}}{\sec^2(y)}$ (60) $\frac{5 \cos(x)}{3e^{3y}}$ (61) $\frac{3 + \sec(x) \tan(x)}{3x + \sec(x)}$ (62) $\sec^2(\pi x^2 - \ln(x)) \left(2\pi x - \frac{1}{x}\right)$

$$\begin{aligned}
(63) \quad & \frac{\cos(x)(4 - \ln(3x + 1)) + \sin(x)\frac{3}{3x+1}}{(4 - \ln(3x + 1))^2} & (64) \quad & \frac{3 \sec^2(x) + 5e^{5x}}{3 \tan(x) + e^{5x}} & (65) \quad & \frac{3 \cos(x)}{2(3 \sin(x) + 1) \sec^2(2y)} \\
(66) \quad & \frac{6x^2}{x^3 + 1} + \frac{3 \sec^2(x)}{\tan(x) + 2} + \frac{\sin(x)}{2(\cos(x) + 2)} & (67) \quad & \frac{2 + \sin(x)}{3(2x - \cos(x))} - \frac{3 \cos(x)}{\sin(x) + 4} - \frac{1}{2(x + 1)} \\
(68) \quad & -4x \tan(x^2 - 1) - \frac{1}{2(x + 3)} - \frac{6x}{x^2 + 1} & (69) \quad & \frac{y^3 \sin(xy) - y \cos\left(\frac{x}{y}\right)}{6y^4 - xy^2 \sin(xy) - x \cos\left(\frac{x}{y}\right)} & (70) \quad & \frac{y^2 \sec^2(xy) - 3y}{\cos(\ln(y)) - xy \sec^2(xy)} \\
(71) \quad & \frac{y \sin\left(\frac{y}{x}\right) - 4x^2}{x^2 \cos(y)e^{\sin(y)} + x \sin\left(\frac{y}{x}\right)} & (72) \quad & \frac{\cos(x)e^{y-\sin(x)} + \frac{4 \cos(y)}{(4x+1)^2}}{e^{y-\sin(x)} - \frac{\sin(y)}{4x+1} + 3} & (73) \quad & 81 \cos(9 - 3x) & (74) \quad & \frac{1}{27} \sin\left(\frac{x}{3}\right) \\
(75) \quad & -9 \sec^2(3x) & (76) \quad & -3 \csc^2(x) & (77) \quad & [-4 \cos(2x) + 4 \sin^2(2x)] e^{\cos(2x)-1} & (78) \quad & 12 \cos(2x) - 12x \sin(2x) \\
(79) \quad & \frac{-4 \sin(2 \ln(x)) - 2 \cos(2 \ln(x))}{x^2} & (80) \quad & -12 \sin(3x) - (18x - 9) \cos(3x) & (81) \quad & \frac{-\sin^2(x) - 4 \sin(x) - 2 \cos^2(x)}{(\sin(x) + 4)^3} \\
(82) \quad & -25 \csc^2(5x) & (83) \quad & -8 & (84) \quad & \frac{-1}{8} \cos\left(\frac{x}{2}\right) & (85) \quad & (x + 1)^{2 \cos(x)} \left[\frac{2 \cos(x)}{x + 1} - 2 \sin(x) \ln(x + 1) \right] \\
(86) \quad & (4 + x)^{\sin(4-x)} \left[\frac{\sin(4-x)}{4+x} - \cos(4-x) \ln(4+x) \right] & (87) \quad & \frac{(6x + 1)^2 \sqrt[4]{2x^2 + 1}}{e^{1-\cos(x)}} \left[\frac{12}{6x + 1} + \frac{x}{2x^2 + 1} - \sin(x) \right] \\
(88) \quad & \frac{\sin(4x) e^{3 \sin(x)}}{\sqrt[3]{9x + 1}} \left[4 \cot(4x) + 3 \cos(x) - \frac{3}{9x + 1} \right] & (89) \quad & \frac{\sqrt{\sin(3x) + 1}}{\cos^2(x) \sqrt[3]{x^2 + 1}} \left[\frac{3 \cos(3x)}{2(\sin(3x) + 1)} + 2 \tan(x) - \frac{2x}{3(x^2 + 1)} \right] \\
(90) \quad & \frac{(x + 2) \sqrt{\cos^3(x)}}{(3x + \cos(2x))^4} \left[\frac{1}{x + 2} - \frac{3}{2} \tan(x) - \frac{12 - 8 \sin(2x)}{3x + \cos(2x)} \right] \\
(91) \quad & \frac{(\sin(3x) - \cos(2x))^4}{2 \sec(x) (\tan(x) + 2)^2} \left[\frac{4(3 \cos(3x) + 2 \sin(2x))}{\sin(3x) - \cos(2x)} - \tan(x) - \frac{2 \sec^2(x)}{\tan(x) + 2} \right] & (92) \quad & (\sin(3x))^{\frac{1}{x+1}} \left[\frac{3 \cot(3x)}{x + 1} - \frac{\ln(\sin(3x))}{(x + 1)^2} \right] \\
(93) \quad & (x^2 + 2)^{\tan(x)} \left[\frac{2x \tan(x)}{x^2 + 2} + \sec^2(x) \ln(x^2 + 2) \right] & (94) \quad & (\tan(2x) + 3)^{\cos(x)} \left[\frac{2 \sec^2(2x) \cos(x)}{\tan(2x) + 3} - \sin(x) \ln(\tan(2x) + 3) \right] \\
(95) \quad & (\sin(3x) + \cos(x))^{\sqrt{x+1}} \left[\frac{(3 \cos(3x) - \sin(x))\sqrt{x+1}}{\sin(3x) + \cos(x)} + \frac{\ln(\sin(3x) + \cos(x))}{2\sqrt{x+1}} \right] \\
(96) \quad & \left(\ln(\cos(x)) + 4 \right)^{\tan(2x)} \left[2 \sec^2(2x) \ln(\ln(\cos(x)) + 4) - \frac{\tan(x) \tan(2x)}{\ln(\cos(x)) + 4} \right] \\
(97) \quad & \frac{\sqrt[3]{3x + 1} e^{\sin(2x)}}{(x^3 + 1)^3} \left[\frac{1}{3x + 1} + 2 \cos(2x) - \frac{9x^2}{x^3 + 1} \right] & (98) \quad & \frac{\cos(2x) \sqrt{4x + 1}}{e^{\sin(3x)}} \left[-2 \tan(2x) + \frac{2}{4x + 1} - 3 \cos(3x) \right] \\
(99) \quad & (\tan(x))^{2x} \left[\frac{2x \sec^2(x)}{\tan(x)} + 2 \ln(\tan(x)) \right] & (100) \quad & \cos(x) & (101) \quad & -2^{54} \cos(2x - 5) & (102) \quad & -3^{75} \cos(3x + 1) \\
(103) \quad & (-2)^{124} \cos(3 - 2x) & (104) \quad & (-6)^{45} \sin(1 - 6x) & (105) \quad & -51 \sin(x) - x \cos(x) \\
(106) \quad & 10! & (107) \quad & 0 & (108) \quad & \frac{53!}{x^{54}} & (109) \quad & -5^{101} e^{-5x} \\
(110) \quad & x = 0, -1/3 & (111) \quad & x = \pm 1/4 & (112) \quad & x = 1, 6 & (113) \quad & x = -3, -2, -1 & (114) \quad & x = -5
\end{aligned}$$