201-103-RE - Supplement G: Applications of the Second Derivative

Find the intervals of concavity, and any points of inflection of the following functions.

(1) $f(x) = \dfrac{3x^3 + 10x - 24}{2x}$

(2) $f(x) = -x^4 + 2x^3 + 5x$

(3) $f(x) = \dfrac{x^3 - x^2 - 8}{x - 1}$

(4) $f(x) = \dfrac{x^3 + 4x + 27}{x}$

(5) $f(x) = x^4 + 4x^3 - 5x$

(6) $f(x) = \dfrac{3}{2}x^2 + \dfrac{12}{x - 1}$

(7) $f(x) = x^4 + 2x^3 - 12x^2$

(8) $f(x) = 4x^5 + 5x^4 - 80x^3$

Use the second derivative test to find the local extrema of the following functions. If the second derivative test fails, state this.

(9) $f(x) = \dfrac{1}{4}x^4 - \dfrac{5}{3}x^3 + 2x^2 + \dfrac{35}{3}$

(10) $f(x) = \dfrac{6x^3 + 96}{x}$

(11) $f(x) = \dfrac{10\left(x^2 + x + 4\right)}{x + 1}$

(12) $f(x) = \dfrac{3}{4}x^4 + 5x^3 + 9x^2 - \dfrac{15}{4}$

(13) $f(x) = \dfrac{x^3 - 54}{x}$

(14) $f(x) = \dfrac{-3\left(x^2 + 2x + 4\right)}{x + 2}$

(15) $f(x) = \dfrac{1}{4}x^4 + x^3 - \dfrac{1}{2}x^2 - 3x$

ANSWERS:

(1) CU: $(-\infty, 0), (2, \infty)$
CD: $(0, 2)$
POI: $(2, 5)$

(2) CU: $(0, 1)$
CD: $(-\infty, 0), (1, \infty)$
POI: $(0, 0), (1, 6)$

(3) CU: $(-\infty, 1), (3, \infty)$
CD: $(1, 3)$
POI: $(3, 5)$

(4) CU: $(-\infty, -3), (0\infty)$
CD: $(-3, 0)$
POI: $(-3, 4)$

(5) CU: $(-2, 0)$
CD: $(-\infty, -2), (0, \infty)$
POI: $(0, 0), (-2, -6)$

(6) CU: $(-\infty, -1), (1, \infty)$
CD: $(-1, 1)$
POI: $(-1, -9/2)$

(7) CU: $(-\infty, -2), (1, \infty)$
CD: $(-2, 1)$
POI: $(-2, -48), (1, -9)$

(8) CU: $(-2.86, 0), (2.10, \infty)$
CD: $(-\infty, -2.86), (0, 2.10)$
POI: $(-2.86, 1433.00), (0, 0), (2.10, -481.74)$

(9) Local min: $(0, 35/3), (4, 1)$
Local max: $(1, 49/4)$

(10) Local min: $(2, 72)$
Local max: none

(11) Local min: $(1, 30)$
Local max: $(-3, -50)$

(12) Local min: $(-3, 3), (0, -15/4)$
Local max: $(-2, 17/4)$

(13) Local min: $(-3, 27)$
Local max: none

(14) Local min: $(-4, 18)$
Local max: $(0, -6)$

(15) Local min: $(-3, -9/4), (1, -9/4)$
Local max: $(-1, 7/4)$