201-203-RE - Practice Set #21: Other Series Tests

Find an expression for the n^{th} partial sum s_n of each of the following telescoping series, and use it to determine whether the series converges or diverges. If a series converges, find its sum.

(1) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

(2) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2-k}$

(3) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k^2+4k+3}$

(4) $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

(5) $\displaystyle\sum_{k=2}^{\infty} \left[\frac{1}{\ln k} - \frac{1}{\ln(k+1)}\right]$

(6) $\displaystyle\sum_{k=1}^{\infty} \left(e^{\frac{1}{k}} - e^{\frac{1}{k+1}}\right)$

(7) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4k^2-1}$

(8) $\displaystyle\sum_{k=3}^{\infty} \frac{3}{k^2+k-2}$

(9) $\displaystyle\sum_{k=2}^{\infty} \left[\sin\left(\frac{\pi}{k}\right) - \sin\left(\frac{\pi}{k+1}\right)\right]$

Use the integral test to determine whether the following series converge or diverge.

(10) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k-2}$

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

(12) $\displaystyle\sum_{k=1}^{\infty} ke^{-k^2}$

(13) $\displaystyle\sum_{n=3}^{\infty} \frac{n^3}{n^4-16}$

(14) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{(k^2+1)^{\frac{3}{2}}}$

(15) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\sqrt{\ln(k)}}$

State whether each of the following series is a geometric series or a p-series, and determine whether the series converges or diverges. Where possible, also find the sum of the series.

(16) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{7^k}$

(17) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^7}$

(18) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}$

(19) $\displaystyle\sum_{k=1}^{\infty} 6^{k+1}$

(20) $\displaystyle\sum_{k=1}^{\infty} k^{-3/4}$

(21) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(22) $2+\frac{2}{5}+\frac{2}{25}+\frac{2}{125}+\cdots$

(23) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots$

(24) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots$

(25) $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\cdots$

(26) $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots$

Use the ratio test to determine whether the following series converge or diverges. If the ratio test is inconclusive, state this.

(27) $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{4^k}$

(28) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

(29) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{4}{3}\right)^n$

(30) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{5^{k+2}}$

(31) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

(32) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(33) $\displaystyle\sum_{n=1}^{\infty} (n+1)5^{-n}$

(34) $\displaystyle\sum_{k=1}^{\infty} \frac{2k!}{k^4}$

(35) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{2^k+3}$

(36) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n+1}$

(37) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{3^k}$

(38) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$

ANSWERS:

(1) $s_n = 1 - \dfrac{1}{n+1}$; converges to 1

(2) $s_n = 1 - \dfrac{1}{n}$; converges to 1

(3) $s_n = \dfrac{5}{6} - \dfrac{1}{n+2} - \dfrac{1}{n+3}$; converges to $\dfrac{5}{6}$

(4) $s_n = -\ln(n+1)$; diverges

(5) $s_n = \dfrac{1}{\ln 2} - \dfrac{1}{\ln(n+1)}$; converges to $\dfrac{1}{\ln 2}$

(6) $s_n = e - e^{\frac{1}{n+1}}$; converges to $e - 1$

(7) $s_n = \dfrac{1}{2} - \dfrac{1}{4n+2}$; converges to $\dfrac{1}{2}$

(8) $s_n = \dfrac{13}{12} - \dfrac{1}{n} - \dfrac{1}{n+1} - \dfrac{1}{n+2}$; converges to $\dfrac{13}{12}$

(9) $s_n = 1 - \sin\left(\dfrac{\pi}{n+1}\right)$; converges to 1

(10) diverges since $\displaystyle\int_1^\infty \frac{dx}{5x-2} = \infty$

(11) converges since $\displaystyle\int_1^\infty \frac{dx}{3^x} = \frac{1}{3\ln 3}$

(12) converges since $\displaystyle\int_1^\infty xe^{-x^2}\ dx = \frac{1}{2e}$

(13) diverges since $\displaystyle\int_3^\infty \frac{n^3}{n^4 - 16}\ dx = \infty$

(14) converges since $\displaystyle\int_1^\infty \frac{x}{(x^2+1)^{3/2}}\ dx = \frac{1}{\sqrt{2}}$

(15) diverges since $\displaystyle\int_2^\infty \frac{dx}{x\sqrt{\ln x}} = \infty$

(16) geometric series with $r = \frac{1}{7}$; converges to $\dfrac{1}{6}$

(17) p-series with $p = 7$; converges

(18) p-series with $p = -\dfrac{1}{2}$; diverges

(19) geometric series with $r = 6$; diverges

(20) p-series with $p = \dfrac{3}{4}$; diverges

(21) p-series with $p = \dfrac{3}{2}$; diverges

(22) geometric series with $r = \dfrac{1}{5}$; converges to $\dfrac{5}{2}$

(23) p-series with $p = 2$; converges

(24) geometric series with $r = \dfrac{1}{4}$; converges to $\dfrac{4}{3}$

(25) geometric series with $r = \dfrac{2}{3}$; converges to 3

(26) p-series with $p = \dfrac{1}{2}$; diverges

(27) diverges

(28) converges

(29) diverges

(30) converges

(31) converges

(32) inconclusive

(33) converges

(34) diverges

(35) diverges

(36) diverges

(37) converges

(38) inconclusive