## 201-203-RE - Practice Set #22: Mixed Series

Determine whether the following series converge or diverges. Justify your answers by referencing a test, and showing why that test may be applied. **In the case of a convergent geometric or telescoping series, find the sum of the series.**

(1) $\displaystyle\sum_{k=2}^{\infty} \frac{5}{\sqrt[4]{k^3}}$

(2) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{6}{7}\right)^{n+1}$

(3) $\displaystyle\sum_{k=1}^{\infty} \ln(k)$

(4) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3n+1}$

(5) $\displaystyle\sum_{n=5}^{\infty} \frac{4^{n-1}}{3n!}$

(6) $\displaystyle\sum_{n=4}^{\infty} \frac{36}{n^2-n-2}$

(7) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{n+1}}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(9) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right)$

(10) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} - \frac{1}{n^4}\right)$

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+3)^{3/2}}$

(12) $\displaystyle\sum_{n=0}^{\infty} n(0.7)^n$

(13) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5+3^{n+1}}$

(14) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.4}}$

(15) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(0.4)^n}$

(16) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}$

(17) $\displaystyle\sum_{n=2}^{\infty} 3^{1+n}5^{1-n}$

(18) $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n 2^n}{n^3}$

(19) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^6+1}}$

(20) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+1}{(n+2)!}$

(21) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k$

(22) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n-2}$

(23) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{3^k} - \frac{1}{3^{k+1}}\right)$

(24) $\displaystyle\sum_{k=3}^{\infty} \frac{4}{k^2+5k+6}$

(25) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{4^k}$

(26) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$

(27) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{-3}{4}\right)^k$

(28) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n+1}$

(29) $\displaystyle\sum_{k=1}^{\infty} \frac{4^{k+2}}{3^{2k-1}}$

(30) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{2n^2-6}$

(31) $\displaystyle\sum_{k=1}^{\infty} \frac{7}{3^k+2}$

(32) $\displaystyle\sum_{k=2}^{\infty} \frac{\ln(k)}{k}$

(33) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(34) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{3^k} + \frac{1}{k^3}\right)$

(35) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

(36) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+3)!}{3!k!4^k}$

(37) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k!}$

(38) $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdots(2n)}{1\cdot 4\cdot 7\cdots(3n-2)}$

(39) $\displaystyle\sum_{k=2}^{\infty} k^{-\frac{2}{3}}$

(40) $\displaystyle\sum_{k=2}^{\infty} \ln\left(1 - \frac{1}{k^2}\right)$

---

**ANSWERS:**

(1) diverges by p-series

(2) converges to 6 by geometric series

(3) diverges by test for divergence

(4) diverges by integral test

(5) converges by ratio test

(6) converges to 13 by telescoping

(7) converges to $\frac{1}{5}$ by geometric series

(8) diverges by test for divergence or ratio test

(9) converges to $-1$ by telescoping

(10) converges by p-series

(11) converges by integral test

(12) converges by ratio test

(13) converges by ratio test

(14) diverges by p-series

(15) diverges by geometric series

(16) converges by integral test

(17) converges to $\frac{27}{2}$ by geometric series

(18) diverges by ratio test

(19) diverges by test for divergence

(20) converges by ratio test

(21) converges to $\frac{1}{3}$ by geometric series

(22) diverges by integral test

(23) converges to $\frac{1}{3}$ by geometric series

(24) converges to $\frac{4}{5}$ by telescoping series

(25) converges by ratio test

(26) diverges by divergence test

(27) converges to $\frac{9}{28}$ by geometric series

(28) diverges by test for divergence

(29) converges to $\frac{192}{5}$ by geometric series

(30) diverges by test for divergence

(31) converges by ratio test

(32) diverges by integral test

(33) diverges by p-series

(34) converges by geometric series and p-series

(35) converges by ratio test

(36) converges by ratio test

(37) converges by ratio test

(38) converges by ratio test

(39) diverges by p-series

(40) converges to $\ln(\frac{1}{2})$ by telescoping series