Math - Calculus II SEQUENCES

PART A : Write the first 4 terms of the following sequences. Determine whether the given sequence converges or diverges. If convergent, find the limit:

(1) $\left\{\dfrac{n+1}{2n-1}\right\} \rightarrow \dfrac{1}{2}$; (2) $\left\{\dfrac{e^n}{n}\right\} \rightarrow \infty$; (3) $\left\{\dfrac{n^2+1}{n}\right\} \rightarrow \infty$

(4) $\left\{\dfrac{\ln n}{n^2}\right\} \rightarrow 0$; (5) $\left\{\left(1+\dfrac{1}{3n}\right)^n\right\} \rightarrow e^{1/3}$

(6) $\left\{\dfrac{n}{n+1} . \sin\dfrac{n\pi}{2}\right\} \rightarrow$ oscillates from 0 to ±1

(7) $a_n = \begin{cases} \dfrac{1+3n}{n} & n \text{ even} \\ \dfrac{1}{n} & n \text{ odd} \end{cases} \rightarrow$ div (odds $\rightarrow$ 0 ; evens $\rightarrow$ 3 as $n \rightarrow \infty$)

(8) $a_n = (-1)^{n+1}\left(\dfrac{2-3n}{4+5n}\right) \rightarrow$ div $\left(\text{odds} \rightarrow -\dfrac{3}{5} \text{ ; evens} \rightarrow \dfrac{3}{5}\right)$

(9) $a_n = \dfrac{2+3n}{4+n} + \dfrac{(-1)^n}{n} \rightarrow 3$

(10) $a_n = \dfrac{2+3n}{4+n} + (-1)^n \rightarrow$ div (evens $\rightarrow$ 4 ; odds $\rightarrow$ 2)

(11) $\{\sin n\pi\} \rightarrow 0$;(12) $\left\{\sin\dfrac{n\pi}{2}\right\} \rightarrow$ div (odds $\rightarrow$ ±1 ; evens $\rightarrow$ 0)

(13) $\{\cos n\pi\} \rightarrow$ div (odds $\rightarrow$ −1 ; evens $\rightarrow$ 1)

(14) $\left\{\dfrac{5^n}{1+5^{2n}}\right\} \rightarrow 0$; (15) $\left\{\dfrac{n^3-1}{n}\right\} \rightarrow \infty$; (16) $\left\{\dfrac{n!}{3^n}\right\} \rightarrow \infty$

(17) $\left\{\dfrac{3^n}{n!}\right\} \rightarrow 0$; (18) $\left\{\dfrac{\ln n}{n}\right\} \rightarrow 0$; (19) $\left\{\dfrac{n}{\ln(n+1)}\right\} \rightarrow \infty$

(20) $\left\{ \dfrac{n}{e^n} \right\} \rightarrow 0$; (21) $\left\{ 1 + (-1)^n \right\} \rightarrow$ div (odds $\to 0$; evens $\to 2$)

(22) $\left\{ \dfrac{n!}{1.3.5.7....(2n-1)} \right\} \rightarrow 0$

PART B : Find a general term for each of the following: (assume that the given pattern continues).

(1) $\{ 2 , 5 , 8 , 11 , 14 , ... \} \rightarrow 3n-1$

(2) $\{ 2 , -5 , 8 , -11 , 14 , ... \} \rightarrow (-1)^{n+1} (3n-1)$

(3) $\{ 1 , -2 , 3 , -4 , 5 , -6 , ... \} \rightarrow (-1)^{n+1}\ n$

(4) $\{ -1 , 3 , -5 , 7 , -9 , ... \} \rightarrow (-1)^n (2n-1)$

(5) $\{ -2 , -7 , -12 , -17 , -22 , ... \} \rightarrow 3-5n$

(6) $\{ 1 , 3 , 9 , 27 , 81 , 243 , ... \} \rightarrow 3^{n-1}$

(7) $\{ 25 , 50 , 100 , 200 , 400 , ... \} \rightarrow 25 (2^{n-1})$

(8) $\{ 4 , 7 , 10 , 13 , 16 , 19 , ... \} \rightarrow 3n+1$

(9) $\{ 2 , 5 , 10 , 17 , 26 , ... \} \rightarrow a_n = \begin{cases} a_1 = 2 \\ a_n = a_{n-1} + (2n-1) \quad n \geq 2 \end{cases}$

(10) $\{ x , x^4 , x^9 , x^{16} , ... \} \rightarrow x^{n^2}$

(11) $\{ 2 , 2 , 2 , 2 , 2 , ... \} \rightarrow a_n = 2$

(12) $\{ 4 , 16 , 64 , 256 , ... \} \rightarrow 4^n$

(13) $\left\{ 1 , -\dfrac{1}{2} , \dfrac{1}{6} , -\dfrac{1}{24} , \dfrac{1}{120} , -\dfrac{1}{720} , ... \right\} \rightarrow (-1)^{n+1} \dfrac{1}{n!}$

(14) $\left\{ \dfrac{10}{9} , -\dfrac{20}{27} , \dfrac{40}{81} , -\dfrac{80}{243} , ... \right\} \rightarrow (-1)^{n+1} \dfrac{10 (2^{n-1})}{3^{n+1}}$