Math - Calculus II TAYLOR & MACLAURIN SERIES

Maclaurin Series: $f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 + \ldots$

Taylor Series: $f(x) = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f^{(3)}(a)}{3!} (x-a)^3 + \frac{f^{(4)}(a)}{4!} (x-a)^4 + \ldots$

Find the Maclaurin series for the following functions.
Express the series in sigma notation and find the interval of convergence.

(1) $f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots = \sum_{n=0}^{\infty} x^n$; $(-1, 1)$ or

$\sum_{n=1}^{\infty} x^{n-1}$

(2) $f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 + \ldots = \sum_{n=0}^{\infty} (-1)^n x^n$; $(-1, 1)$ or

$\sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1}$

(3) $f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$; $(-\infty, \infty)$ or

$\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$

(4) $f(x) = e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$; $(-\infty, \infty)$ or

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n-1}}{(n-1)!}$

(5) $f(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$; $(-\infty, \infty)$

(6) $f(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$; $(-\infty, \infty)$

(7) $f(x) = \arctan x = x - \frac{2!}{3!}x^3 + \frac{4!}{5!}x^5 - \frac{6!}{7!}x^7 + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n)!\,x^{2n+1}}{(2n+1)!}$; $(-1,\ 1)$

(8) $f(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots = \sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{x^n}{n}$; $(-1,\ 1]$

then $\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$, the alternating harmonic series

Find the Taylor series for the following functions.
Express the series in sigma notation and find the interval of convergence.

(1) $f(x) = \frac{1}{x}$ centered at a = 1

$$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - (x-1)^5 + \ldots = \sum_{n=0}^{\infty} (-1)^n\,(x-1)^n \quad ; \quad (0,\ 2)$$

(2) $f(x) = \ln(x)$ centered at a = 1

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \ldots = \sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{(x-1)^n}{n} \quad ; \quad (0,\ 2]$$

(3) $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$

$$1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} - \frac{\left(x-\frac{\pi}{2}\right)^6}{6!} + \ldots = \sum_{n=0}^{\infty} (-1)^n\,\frac{\left(x-\frac{\pi}{2}\right)^{2n}}{(2n)!} \quad ; \quad (-\infty,\ \infty)$$

(4) $f(x) = e^x$ centered at a = 3

$$e^3 + \frac{e^3(x-3)}{1!} + \frac{e^3(x-3)^2}{2!} + \frac{e^3(x-3)^3}{3!} + \ldots = \sum_{n=0}^{\infty} e^3\,\frac{(x-3)^n}{n!} \quad ; \quad (-\infty,\ \infty)$$

(5) $f(x) = \cos x$ centered at $a = \pi$

$$-1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!} - \ldots = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi)^{2n}}{(2n)!} \quad ; \quad (-\infty,\ \infty)$$

(6) $f(x) = \cos x$ centered at $a = \frac{\pi}{2}$

$$-\frac{\left(x-\frac{\pi}{2}\right)}{1!} + \frac{\left(x-\frac{\pi}{2}\right)^3}{3!} - \frac{\left(x-\frac{\pi}{2}\right)^5}{5!} + \ldots = \sum_{n=1}^{\infty} (-1)^n \frac{\left(x-\frac{\pi}{2}\right)^{2n-1}}{(2n-1)!} \quad ; \quad (-\infty,\ \infty)$$

(7) $f(x) = \sqrt{1+x}$ centered at a = 0 ; write only the first 5 terms

$$1 + \frac{x}{2} - \frac{x^2}{(4)(2!)} + \frac{3x^3}{(8)(3!)} - \frac{3x^5}{(16)(4!)} + \ldots$$