Math - Calculus II   <u>TRIGONOMETRIC SUBSTITUTION</u>

Integrals containing:

a^2+u^2 ($u = a\tan\theta$) ; a^2-u^2 ($u = a\sin\theta$) ; u^2-a^2 ($u = a\sec\theta$)

Note: Always try <u>direct substitution</u> or <u>algebraic substitution</u> <u>first</u> - where applicable these techniques are easier and quicker to apply.

Determine:

(1) $\int \frac{dx}{\sqrt{1-4x^2}}$; (2) $\int \frac{dx}{x^2+25}$; (3) $\int \frac{x\,dx}{x^4+16}$; (4) $\int \frac{dx}{\sqrt{2-5x^2}}$

(5) $\int \frac{3\,dx}{x\sqrt{x^2-9}}$; (6) $\int \frac{r\,dr}{\sqrt{16-9r^4}}$; (7) $\int \frac{dx}{x\sqrt{16x^2-9}}$; (8) $\int \frac{e^x\,dx}{7+e^{2x}}$

(9) $\int \frac{\sin x\,dx}{\sqrt{2-\cos^2 x}}$; (10) $\int \frac{dx}{\sqrt{x}\,(1+x)}$; (11) $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$

(12) $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}}$; (13) $\int_{\frac{1}{\sqrt{2}}}^{1} \frac{dx}{x\sqrt{4x^2-1}}$; (14) $\int_1^{e} \frac{dx}{x\,(1+(\ln x)^2)}$

(15) $\int_0^1 \frac{1+x}{1+x^2}\,dx$; (16) $\int \frac{dx}{x^2\sqrt{4-x^2}}$; (17) $\int \frac{dx}{x\sqrt{x^2+4}}$;

(18) $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

(19) $\int \frac{dx}{x\sqrt{25-x^2}}$; (20) $\int \frac{dx}{\sqrt{x^2-a^2}}$; (21) $\int \sqrt{1-x^2}\,dx$; (22) $\int \frac{dw}{w^2\sqrt{w^2-7}}$

(23) $\int x^2\sqrt{16-x^2}\,dx$; (24) $\int \frac{dx}{(x^2+4)^{3/2}}$; (25) $\int \frac{dx}{(4x^2-9)^{3/2}}$

(26) $\int \frac{dx}{x^4\sqrt{16+x^2}}$; (27) $\int \frac{dx}{x\sqrt{x^4-4}}$; (28) $\int \frac{\sec^2 x\,dx}{(4-\tan^2 x)^{3/2}}$

(29) $\int \frac{e^{-x}\,dx}{(9e^{-2x}+1)^{3/2}}$; (30) $\int \frac{x^3\,dx}{\sqrt{16-x^2}}$; (31) $\int_0^4 \frac{dx}{(16+x^2)^{3/2}}$

(32) $\int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2 \sqrt{x^2+9}}$; (33) $\int_0^1 \frac{x^2\, dx}{\sqrt{4-x^2}}$; (34) $\int_4^6 \frac{dx}{x \sqrt{x^2-4}}$

(35) $\int_4^8 \frac{dw}{(w^2-4)^{3/2}}$; (36) $\int_0^5 x \sqrt{25-x^2}\, dx$; (37) $\int \frac{dx}{x^4 \sqrt{x^2+3}}$

(38) $\int \frac{\sqrt{x^2-25}}{x}\, dx$

Answers:

(1) $\frac{1}{2} \arcsin 2x + C$; (2) $\frac{1}{5} \arctan\left(\frac{x}{5}\right) + C$; (3) $\frac{1}{8} \arctan\left(\frac{x^2}{4}\right) + C$

(4) $\frac{1}{\sqrt{5}} \arcsin\left(\frac{\sqrt{5}x}{\sqrt{2}}\right) + C$; (5) $\operatorname{arcsec}\left(\frac{x}{3}\right) + C$; (6) $\frac{1}{6} \arcsin\left(\frac{3r^2}{4}\right) + C$

(7) $\frac{1}{3} \operatorname{arcsec}\left(\frac{4x}{3}\right) + C$; (8) $\frac{1}{\sqrt{7}} \arctan\left(\frac{e^x}{\sqrt{7}}\right) + C$;

(9) $-\arcsin\left(\frac{\cos x}{\sqrt{2}}\right) + C$; (10) $2 \arctan \sqrt{x} + C$; (11) $\frac{\pi}{6}$; (12) 1

(13) $\frac{\pi}{12}$; (14) $\arctan(\ln x) + C$; (15) $\frac{\pi}{4} + \frac{1}{2}\ln(2)$; (16) $\frac{-\sqrt{4-x^2}}{4x} + C$

(17) $\frac{1}{2} \ln\left|\frac{\sqrt{x^2+4} - 2}{x}\right| + C$; (18) $-\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C$

(19) $\frac{1}{5} \ln\left|\frac{5 - \sqrt{25-x^2}}{x}\right| + C$; (20) $\ln\left|\frac{x + \sqrt{x^2-a^2}}{a}\right| + C$ or $\ln\left| x + \sqrt{x^2-a^2} \right| + C$

(21) $\frac{1}{2} \arcsin x + \frac{x\sqrt{1-x^2}}{2} + C$; (22) $\frac{\sqrt{w^2-7}}{7w} + C$

(23) $32 \arcsin\left(\frac{x}{4}\right) - \frac{x\sqrt{16-x^2}}{4}(8-x^2) + C$; (24) $\frac{x}{4\sqrt{x^2+4}} + C$; (25) $\frac{-x}{9\sqrt{4x^2-9}} + C$

(26) $\dfrac{-1}{4^4(3)}\left(\dfrac{\sqrt{16+x^2}}{x}\right)^3 + \dfrac{1}{4^4}\dfrac{\sqrt{16+x^2}}{x} + C = \dfrac{(x^2-8)\sqrt{16+x^2}}{384x^3} + C$

(27) $\dfrac{1}{4}\operatorname{arcsec}\left(\dfrac{x^2}{2}\right) + C$; (28) $\dfrac{\tan x}{4\sqrt{4-\tan^2 x}} + C$

(29) $\dfrac{-e^{-x}}{\sqrt{9e^{-2x}+1}} + C$; (30) $-24\sqrt{3} + \dfrac{128}{3}$; (31) $\dfrac{1}{16\sqrt{2}}$

(32) $\dfrac{2}{9} - \dfrac{2\sqrt{3}}{27}$; (33) $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}$; (34) $\dfrac{1}{2}\operatorname{arcsec}(3) - \dfrac{\pi}{6}$

(35) $\dfrac{1}{2\sqrt{3}} - \dfrac{1}{\sqrt{15}}$; (36) $\dfrac{125}{3}$; (37) $-\dfrac{(x^2+3)^{3/2}}{27x^3} + \dfrac{\sqrt{x^2+3}}{9x} + C$ or $\dfrac{(2x^2-3)\sqrt{x^2+3}}{27x^3} + C$

(38) $\sqrt{x^2-25} - 5\operatorname{arcsec}\left(\dfrac{x}{5}\right) + C$