Math - Calculus II

TRIG SUBSTITUTION or SUBSTITUTION or ALGEBRAIC SUBSTITUTION

Determine:

( a) $\displaystyle\int \frac{x\,dx}{\sqrt{x^2-1}}$ ; ( b) $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{(4-x^2)^{3/2}}$ ; ( c) $\displaystyle\int \frac{x^3\,dx}{\sqrt{4-x^2}}$ ; ( d) $\displaystyle\int \frac{x^2\,dx}{(9-4x^2)^{3/2}}$

( e) $\displaystyle\int \frac{\sqrt{9x^2-4}}{x}\,dx$ ; ( f) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-1}}$ ; ( g) $\displaystyle\int x^3\sqrt{x^2-4}\,dx$

( h) $\displaystyle\int \frac{x^3\,dx}{\sqrt{4x^2-9}}$ ; ( i) $\displaystyle\int_{\sqrt{5}}^{\sqrt{20}} \frac{dx}{(x^2-4)^{3/2}}$ ; ( j) $\displaystyle\int_0^{3/2} \frac{dx}{\sqrt{9-2x^2}}$ ; ( k) $\displaystyle\int \frac{3x+4}{x^2+1}\,dx$

Answers:

( a) subs: $\sqrt{x^2-1} + C$ ; ( b) trig subs: $\dfrac{\sqrt{3}}{4}$ ;

( c) alg subs: $\dfrac{1}{3}(4-x^2)^{3/2} - 4\sqrt{4-x^2} + C$

( d) trig subs: $\dfrac{x}{4\sqrt{9-4x^2}} - \dfrac{1}{8}\arcsin\left(\dfrac{2x}{3}\right) + C$

( e) trig subs: $\sqrt{9x^2-4} - 2\,\mathrm{arcsec}\left(\dfrac{3x}{2}\right) + C$

( f) trig subs: $\dfrac{\sqrt{x^2-1}}{x} + C$

( g) alg subs: $\dfrac{4}{3}(x^2-4)^{3/2} + \dfrac{1}{5}(x^2-4)^{5/2} + C$

( h) alg subs: $\dfrac{9}{16}\sqrt{4x^2-9} + \dfrac{1}{48}(4x^2-9)^{3/2} + C$ ; ( i) trig subs: $\dfrac{\sqrt{5}}{8}$

( j) trig subs: $\dfrac{\pi}{4\sqrt{2}}$ ; ( k) split into 2 integrals: $\dfrac{3}{2}\ln(x^2+1) + 4\arctan x + C$

Math - Calculus II                                    PARTIAL FRACTIONS
Ex 7.4 p 498 (Stewart)

$$\int \frac{\text{polynomial}}{\text{polynomial}}\,dx = \int \frac{P(x)}{Q(x)}\,dx$$ ; degree of numerator < degree of denominator

otherwise long division first ; then factor the denominator
Case I: factors of denominator are all linear and raised to the first power

example: $$\frac{x^2+2}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

solve for A, B, C and the integration is easy.
Case II: factors of denominator are all linear but some of the factors are raised to a power higher than one.

Example: $$\frac{x^2+2}{x^3(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+2}$$ all powers between 1 and 2 must be included.
Again, solve for A, B, C, D and the resulting integrals are easy to solve.

Ex 7.4 : 1-7, 13-27, 39, 51( $u = e^x$ ), 52( $u = \sin x$ ) ($4^{th}$ edition)

Ex 7.4 : 1-4 , 7-28 , 35 ($5^{th}$ edition)