Lines in $\mathbb{R}^2$ and $\mathbb{R}^3$

(1) Find the equation of the line passing through (1 , -4) and (3 , 7)
(a) in standard form $(\mathrm{A}x+\mathrm{B}y=\mathrm{C})$; (b) in vector form ; (c) in parametric form.

(2) Find the equation of the line passing through (1 , -5) with slope = -2/3
(a) in standard form ; (b) in vector form ; (c) in parametric form

(3) Find the equation of the line L which :
(a) parallel to (2 , -1 , 0) and passing through $P(1,-1,3)$

(b) passing through $P(3,-1,4)$ and $Q(1,0,-1)$

(c) passing through $P(3,-1,4)$ and $Q(3,-1,5)$

(d) parallel to (1 , 2 , -7) and passing through (0 , 0 , 0)

(e) passing through $P(1,\ 0,\ -3)$ and parallel to the line : $\begin{cases} x = -1 + 2t \\ y = 2 - t \\ z = 3 + 3t \end{cases}$

(f) passing through $P(2,-1,1)$ and parallel to the line $(x,y,z)=(2,1,0)+t(-1,0,1)$

Answers:

(1) (a) $11x-2y=19$; (b) $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix}+t\begin{pmatrix}2\\11\end{pmatrix}$ or $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3\\7\end{pmatrix}+t\begin{pmatrix}2\\11\end{pmatrix}$; (c) $\begin{cases}x=1+2t\\y=-4+11t\end{cases}$ or $\begin{cases}x=3+2t\\y=7+11t\end{cases}$

(2) (a) $2x+3y=-13$; (b) $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\-5\end{pmatrix}+t\begin{pmatrix}3\\-2\end{pmatrix}$; (c) $\begin{cases}x=1+3t\\y=-5-2t\end{cases}$; (3 a) $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-1\\3\end{pmatrix}+t\begin{pmatrix}2\\-1\\0\end{pmatrix}$

(3 b) $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-1\\4\end{pmatrix}+t\begin{pmatrix}2\\-1\\5\end{pmatrix}$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+t\begin{pmatrix}2\\-1\\5\end{pmatrix}$;

(3 c) $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-1\\4\end{pmatrix}+t\begin{pmatrix}0\\0\\1\end{pmatrix}$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-1\\5\end{pmatrix}+t\begin{pmatrix}0\\0\\1\end{pmatrix}$; (3 d) $(x,y,z)=t(1,2,-7)$

(3 e) $(x,y,z)=(1,0,-3)+t(2,-1,3)$

(3 f) $(x,y,z)=(2,-1,1)+t(-1,0,1)$

(4) Verify that P and Q lie on the given line and R does not lie on the line.

(a) $l:(x,y,z)=(3,2,1)+t(-4,1,-1)$
$P(-1,3,0)$; $Q(11,0,3)$; $R(-1,4,5)$

(b) $l:(x,y,z)=(4,3,1)+t(-1,0,-2)$
$P(2,3,-3)$; $Q(-1,3,-9)$; $R(2,3,7)$

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(5) Find the intersection (if any) of the following lines.

(a) $l_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\quad l_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

(b) $l_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ $\quad l_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

(c) $l_1 : (x, y, z) = (3, -1, 2) + t\ (1, 1, -1)$ $\quad l_2 : (x, y, z) = (1, 1, -2) + s\ (2, 0, 3)$

(d) $l_1 : (x, y, z) = (4, -1, 5) + t\ (1, 0, 1)$ $\quad l_2 : (x, y, z) = (2, -7, 12) + s\ (0, -2, 3)$

(e) $l_1 : (x, y, z) = (4, -1, 5) + t\ (1, 2, 3)$ $\quad l_2 : (x, y, z) = (-1, 2, 5) + s\ (2, 4, 6)$

(f) $l_1 : (x, y, z) = (4, -1, 5) + t\ (1, 2, 3)$ $\quad l_2 : (x, y, z) = (5, 1, 8) + s\ (2, 4, 6)$

<u>Answers</u>:
(5) (a) unique solution (2, 3, 0) ; (b and c) skew lines (no intersection, d's not multiples of each other)
(d) unique solution (2, -1, 3) ; (e) parallel lines (no intersection and d's are multiples)
(f) parametric solution for s and t $\rightarrow l_1$ and l_2 are the same line.

Text: Ex 3.5 ; 9 , 10 , 15 , 18 , 19 , 35 (change one of the t's to an s)