Matrix Operations

(1) Find a , b , c , d if

$$(a)\ \begin{bmatrix} a-b & b-c \\ c-d & d-a \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix} ;\ (b)\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} b & c \\ d & a \end{bmatrix} ;\ (c)\ \begin{bmatrix} a+b & 2c+d \\ a-b & 3c-a \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answers: (a) $(a,b,c,d)=(-2+t\,,-4+t\,,-6+t\,,t)$; (b) $(a,b,c,d)=(t\,,t\,,t\,,t)$; i.e. $a=b=c=d$

(c) $(a,b,c,d)=(2\,,-1\,,2\,,-2)$

(2)

$$(a)\ 3\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 5\begin{bmatrix} 6 \\ 2 \end{bmatrix} + 7\begin{bmatrix} 1 \\ -1 \end{bmatrix} ;\ (b)\ \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}^t ;\ (c)\ 3\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}^t - 2\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Answers: (a) $\begin{bmatrix} -14 \\ -20 \end{bmatrix}$; (b) $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 4 \\ 2 & -4 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} 4 & -1 \\ -1 & -6 \end{bmatrix}$

(3) Find A if (a) $5A - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = 3A - \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}$; (b) $3A + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 5A - 2\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Answers: (a) $A = \begin{bmatrix} -2 & -1 \\ -2 & 1 \end{bmatrix}$; (b) $A = \begin{pmatrix} 4 \\ 1/2 \end{pmatrix}$

(4) Find A in terms of B if (a) $A+B=3A+2B$; (b) $2A-B=5(A+2B)$

Answers: (a) $A=-\frac{1}{2}B$; (b) $A=-\frac{11}{3}B$

(5) Simplify : Assume A , B and C are matrices of the same size.

(a) $2\big(9(A-B)+7(2B-A)\big)-2\big(3(2B+A)-2(A+3B)-5(A+B)\big)$

(b) $5\big[3(A-B+2C)-2(3C-B)-A\big]+2\big[3(3A-B+C)+2(B-2A)-2C\big]$

Answers: (a) $12A+20B$; (b) $20A-7B+2C$

(6) Find s and t such that $A = A^t$

(a) $A = \begin{pmatrix} 1 & s \\ -2 & t \end{pmatrix}$; (b) $A = \begin{pmatrix} s & t \\ st & 1 \end{pmatrix}$; (c) $A = \begin{pmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{pmatrix}$; (d) $A = \begin{pmatrix} 2 & s & t \\ 2s & 0 & s+t \\ 3 & 3 & t \end{pmatrix}$; (e) $A = \begin{pmatrix} s & s^2 \\ st & t \end{pmatrix}$

Answers :

(a) $s=-2$, t = any real number ; (b) $s=1$, t = any real number $\to \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}$ or $t=0$, s = any real number $\to \begin{bmatrix} s & 0 \\ 0 & 1 \end{bmatrix}$

(c) $s=0, t=0$ or $s=1, t=2$; (d) $s=0, t=3$; (e) $s=0$, t = any real number or $s=t$ = any real number

Matrix Operations

(7) Find A given:

$$(a)\ \left(A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix} \right)^t = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix} ;\ (b)\ \left(3A^t + 2\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right)^t = \begin{pmatrix} 8 & 0 \\ 3 & 1 \end{pmatrix}$$

$$(c)\ \left(2A - 3\begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \right)^t = 3A^t + \begin{pmatrix} 2 & 1 & -1 \end{pmatrix}^t$$

$$(d)\ \left(2A^t - 5\begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix} \right)^t = 4A - 9\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

Answers: $(b)\ A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$; $(c)\ A = \begin{pmatrix} -5 & -7 & 1 \end{pmatrix}$; $(d)\ A = \begin{pmatrix} 7 & 7 \\ -\frac{9}{2} & -5 \end{pmatrix}$

(8) Find a , b , c , d if

$$(a)\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix} ;\ (b)\ \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -1 & 4 \end{pmatrix}$$

Answer: $(b)\ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}$

(9) Find all possible matrix products $A^2, AB, AC, BA, CA, BC, CB, C^2, B^2$ for

$(a)\ A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 0 \end{pmatrix}$; $B = \begin{pmatrix} 1 & -2 \\ 1/2 & 3 \end{pmatrix}$; $C = \begin{pmatrix} -1 & 0 \\ 2 & 5 \\ 0 & 3 \end{pmatrix}$; $(b)\ A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}$; $B = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}$; $C = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}$

Answers: $(b)\ BA = \begin{pmatrix} -1 & 4 & -10 \\ 1 & 2 & 4 \end{pmatrix}$, $B^2 = \begin{pmatrix} 7 & -6 \\ -1 & 6 \end{pmatrix}$, $CB = \begin{pmatrix} -2 & 12 \\ 2 & -6 \\ 1 & 6 \end{pmatrix}$, $AC = \begin{pmatrix} 4 & 10 \\ -2 & -1 \end{pmatrix}$, $CA = \begin{pmatrix} 2 & 4 & 8 \\ -1 & -1 & -5 \\ 1 & 4 & 2 \end{pmatrix}$

(10) Verify that $A^2 - A - 6I = 0$ if $(a)\ A = \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix}$; $(b)\ A = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

(11) Given $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \end{pmatrix}$; $C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 5 & 8 \end{pmatrix}$; $D = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & 5 \end{pmatrix}$

Verify that $(a)\ A(BC) = (AB)C$; $(b)\ (CD)^t = D^t C^t$

Matrix Operations

(12) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(a) if $AB = BA$ where $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, show that $d = a, c = 0$ so that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$

(b) if $AB = BA$ where $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, show that $d = a, b = 0$ so that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & a \end{pmatrix}$

(13) Find A_{2x2} such that : (a) $A^2 = 0$ and $A \neq 0$; (b) $A^2 = I$ and $A \neq I$

(14) Given $U = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $UA = 0$; prove $A = 0$ ($a = b = c = d = 0$)

(15) Simplify (assume A , B , C , D are square matrices) :

(a) $A(3B - C) + (A - 2B)C + 2B(C + 2A)$; (b) $A(B + C - D) + B(C - A + D) - (A + B)C + (A - B)D$
(c) $AB(BC - CB) + (CA - AB)BC + CA(A - B)C$; (d) $(A - B)(C - A) + (C - B)(A - C) + (C - A)^2$

Answers: (b) $AB - BA$; (d) 0

(16) Use A_{2x2} , B_{2x2} (examples using numbers) to illustrate the following :

(a) $(A + B)^2 \neq A^2 + 2AB + B^2$; (b) $(A - B)(A + B) \neq A^2 - B^2$
(c) $AB = AC$ does not imply that $B = C$; (d) $AB = 0$ does not imply $A = 0$ or $B = 0$