Question 1: (*12 pts*) For each of the following functions, find the derivative $\dfrac{dy}{dx}$. *You do not have to simplify your answers.*

a) $y = \ln\left(\dfrac{x^4 \cos^6(3x+2)}{\sqrt[3]{x^2+1}}\right)$

b) $y = \sec^3(e^{4x-1})$

c) $3\cot(x+y) = \sin(y^2)$

d) $y = e^{\left(\frac{1}{x^2}\right)} \cos^{-1}(\sqrt{x})$

Question 2: (*8 pts*) For each of the following functions, find the second derivative $\dfrac{d^2y}{dx^2}$. *Simplify your answers as much as possible.*

a) $y = x^2 \tan\left(\dfrac{1}{x}\right)$

b) $y = x\sin(\ln(x))$

Question 3: (*4 pts*) Find the equation of the line that is tangent to the graph of $x^2 + y^2 = 25$, at the point (3,-4).

Question 4: (*4 pts*) Find the slope of the line that is normal to the graph of $y = \dfrac{6}{(x^2+1)^2}$ at $x = 1$.

Question 5: (*5 pts*) Solve the equation $\cos(2x) - 3x = -1$, using Newton's method. Give an answer that is accurate to four decimals, and start with a guess of $x_1 = 1$.

Question 6: (*4 pts*) The impedance Z (in Ω) in an electric circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. If $R = 2500\ \Omega$ and $X_L = 1500\ \Omega$, then find the value of X_C that makes the impedance Z a minimum.

Question 7: (*4 pts*) The electrical potential on the line $3y - x = 2$ is given by the function $V = 4x^2 - 18y^2 + 2$. At what point of the line is the potential minimum?

Question 8: (*30 pts*) Evaluate the following integrals.

a) $\displaystyle\int \frac{1}{x^4} - x^4 + e^x - \frac{1}{e^4}\,dx$

b) $\displaystyle\int \sin^{-1}(x)\,dx$

c) $\displaystyle\int x^2\, e^{x^3}\,dx$

d) $\displaystyle\int 3x^2\, \ln(x)\,dx$

e) $\displaystyle\int x^2\, \sin(2x)\,dx$

f) $\displaystyle\int \frac{-3}{\sqrt{4-9x^2}}\,dx$

g) $\displaystyle\int \frac{\tan(\ln(x))}{x}\,dx$

h) $\displaystyle\int \frac{\cos(x)}{3+\sin^2(x)}\,dx$

i) $\displaystyle\int_1^5 \left(\sqrt{2x-1}\right)^3\,dx$

j) $\displaystyle\int \frac{x+2}{x^2+4x+5}\,dx$

Question 9: (*4 pts*) Find the area enclosed by the curves $y = x^2$ and $y = 2 - x$.

Question 10: (*5 pts*) Give an estimate of $\int_0^1 \sqrt{x^3 + 1}\, dx$ to four decimals, using $n = 4$ and

a) the Trapezoidal Rule **b)** Simpson's Rule

Question 11: (*4 pts*) In coming to a stop, the acceleration of a car is given by $a(t) = -4t$. The car is traveling at 32 m/s when it starts braking.

a) How long does it take for the car to stop?

b) What is the car's braking distance?

Question 12: (*4 pts*) Find a_0 and b_3 of the Fourier series for the function

$$f(x) = \begin{cases} 0 & \text{if } -\pi \leqslant x < 0 \\ x & \text{if } 0 \leqslant x < \pi \end{cases}$$

Question 13: (*2 pts*) Determine if the function $y = x^4 + x + C\ln(x)$ is a solution of the differential equation $xy'' + y' = 16x^3$.

Question 14: (*4 pts*) Find the solution of the differential equation $y' = (1-y)\cos(x)$, with the condition that $y = 0$ when $x = \dfrac{\pi}{6}$.

Question 15: (*6 pts*) Find a general solution of the following differential equations.

a) $y' = \sin(x)\sec(y)$

b) $y' - y = 3x$

Answers

1. a) $y' = \dfrac{4}{x} - 18\tan(3x+2) - \dfrac{2x}{3(x^2+1)}$ **b)** $y' = 12\sec^3(e^{4x-1})\tan(e^{4x-1})$

c) $y' = \dfrac{-3\csc^2(x+y)}{3\csc^2(x+y) + 2y\cos(y^2)}$ **d)** $y' = -e^{1/x^2}\left(\dfrac{1}{2\sqrt{1-x}\sqrt{x}} + \dfrac{2\cos^{-1}(\sqrt{x})}{x^3}\right)$

2. a) $y' = 2x\tan\left(\dfrac{1}{x}\right) - \sec^2\left(\dfrac{1}{x}\right)$ **b)** $y' = \sin(\ln(x)) + \cos(\ln(x))$

3. $y = \dfrac{3}{4}x - \dfrac{25}{4}$

4. $1/3$

5. 0.5086

6. $X_C = 1500$

7. (2,4/3)

8. **a)** $\dfrac{-1}{3x^3} - \dfrac{x^5}{5} + e^x - e^{-4}x + C$ **b)** $x\sin^{-1}(x) + \sqrt{1-x^2} + C$ **c)** $\dfrac{e^{x^3}}{3} + C$
d) $x^3\ln(x) - \dfrac{x^3}{3} + C$ **e)** $\dfrac{-x^2\cos(2x)}{2} + \dfrac{x\sin(2x)}{2} + \dfrac{\cos(2x)}{4} + C$
f) $-\sin^{-1}\left(\dfrac{3x}{2}\right) + C$ **g)** $-\ln(\cos(\ln(x))) + C$ **h)** $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{\sin(x)}{\sqrt{3}}\right) + C$
i) $\dfrac{242}{5}$ **j)** $\dfrac{1}{2}\ln|x^2+4x+5| + C$

9. 9/2

10. **a)** 1.1170 **b)** 1.1114

11. **a)** 4s **b)** 256/3 m

12. $a_0 = \dfrac{\pi}{4}$ $b_3 = \dfrac{1}{3}$

13. No

14. $y = 1 - e^{1/2-\sin(x)}$

15. **a)** $y = \sin^{-1}(-\cos(x) + C)$ **b)** $y = -3x - 3 + Ce^x$