MATH 201-DDD-05 - May 2010 Exam with Solutions

1. A jury of 15 people is to be selected from a panel consisting of 10 men and 12 women.

[2] (a) How many different jury selections are possible?

[2] (b) How many different jury selections contain exactly 11 women?

[3] (c) If the choice is made randomly, what is the probability that the jury contains at most 10 women?

2. An urn contains 6 marbles; 3 red and 3 green. The following experiment is conducted. Marbles are randomly drawn one at a time from the urn and kept aside until a red marble is drawn out. Let X denote the number of green marbles drawn out from such an experiment.

[4] (a) Use a table to describe the probability mass function of X?

[2] (b) What is $E(X)$?

3. Suppose you roll a fair die and then flip a number of fair coins equal to the number on the die. (So if the die shows 3 you flip 3 coins.)

[4] (a) What is the probability of getting 5 heads?

[2] (b) What is the conditional probability that the die landed with 5 facing up if the number of heads is 5?

[6] 4. In a major city 10% of cars have an alarm system installed. A car without an alarm has a 0.02 probability of being stolen, whereas a car with an alarm has the probability of being stolen cut in half. If an car from this major city is stolen what is the probability that it had an alarm system installed?

5. Customers at a particular store pay by cash 20% of the time, by debit 50% of the time and credit card the remainder of the time. Assuming that a customer's method of payment is independent of others, find the following probabilities.

[2] (a) That the next three customers all pay by different methods.

[2] (b) That at least one of next three customers pay by cash.

[2] (c) Given that at least one of the next three customers pay by cash that they are all pay by cash.

6. The number of homes sold by the Acme Realty company follows a Poisson distribution with an average of 2 homes per day.

[2] (a) What is the probability that more than 3 homes will be sold tomorrow?

[2] (b) What is the probability that in the following work week (5 days), exactly 15 homes?

7. A certain café sells coffee in two sizes; medium and large. 40% of its customers buy medium and 60% buy large. Consider randomly selecting 25 customers who buy coffee.

[4] (a) What are the mean value and standard deviation of the number who want a large coffee.

[3] (b) What is the probability that the number who want a large coffee is greater than 1 standard deviation from the mean.

[3] (c) Suppose that a medium coffee cost \$1.50 and a large cost \$2.00, what is the expected revenue from coffee from the 25 customers.

[5] 8. What is the approximate probability that if a fair die is rolled 300 times that six will face up at least 60 times?

9. The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 40V and standard deviation 1.5 V.

[3] (a) What is the probability that the voltage of a single diode is between 39 and 42?

[4] (b) What value is such that only 15% of all diodes have voltages exceeding that value?

[2] (c) If four diodes are independently selected, what is the probability that at least one has a voltage between 39 and 42?

[2] 10. Describe what a type II error is.

11. A random sample of 100 men was used to calculate the 95% confidence interval (173.81, 174.49) for the mean height (in cm) of men.

[1] (a) What was the sample mean?

[1] (b) What was the sample standard deviation?

[4] 12. A random sample of 23 people were given a typhoid shot. Regular blood tests showed that the shot provided protection (for the sample) for a mean time of 36 months with a standard deviation 1.9 months. Find a 90% confidence interval for the standard deviation of population protection time.

[6] 13. A random sample of 350 adults found that 235 had less than seven hours of sleep each night of the workweek. At the 0.05 level of significance does this evidence support the claim that 61% of adults sleep less than seven hours each night of the workweek? Use the P-value method to answer the question.

14. The Sit and Reach Test is a popular measure of flexibility. It measure how far a person can reach past their feet when they are sitting on the floor with their legs straight out. The Sit and Reach Test results (in cm) of 8 randomly selected men are as follows:

$$4 \quad 5 \quad -2 \quad 6 \quad -5 \quad 10 \quad -5 \quad 6$$

Assume that test results for this Test are normally distributed.

[4] (a) Use this sample to create a 95% confidence interval for average Sit and Reach Test result for men.

[5] (b) Does this sample provide evidence that the true average Sit and Reach Test result for men is less than 5 cm? (Do a hypothesis test at significance level of .05)

[6] 15. The following data values were obtained from an expirement designed to estimate the reduction in diastolic blood pressure as a result of consuming a salt-free diet for two weeks. Assume diastolic readings to be normally distributed.

Before	93	106	87	92	102	95	88	110
After	92	102	89	92	101	96	88	105

Does the data suggest the diet reduces diastolic blood pressure by more than 2 units? Do a test at level .05.

[6] 16. The body temperatures (in Celcius) of a random sample of women and a random sample of men are summarized in the table.

Gender	Sample size	Sample mean	Sample SD
Female	70	36.88	.413
Male	65	36.72	.388

Does this data suggest that women's mean body temperature is higher then men's. Do a level .01 test using a rejection region.

[6] 17. Criminologists have long debated whether there is a relationship between weather conditions and the incidence of violent crime. A study classified 1400 homicides according to season, resulting in the accompanying data. Using $\alpha = .01$ test the null hypothesis of equal proportions by using the chi-squared table to say as much as possible about the P-value.

Winter	Spring	Summer	Fall
338	345	382	335

SOLUTIONS

1. (a) $\binom{22}{15} = \mathbf{170\,544}$

   (b) $\binom{12}{11}\binom{10}{4} = 12\binom{10}{4} = \mathbf{2\ 520}$

   (c)

$$\begin{aligned}\text{P(at most 10 women)} &= 1 - \text{P(at least 11 women)}\\ &= 1 - \frac{12\binom{10}{4} + \binom{10}{3}}{\binom{22}{15}}\\ &= \mathbf{.9845} \text{ (approx.)}\end{aligned}$$

2. (a)

x	0	1	2	3
$p(x)$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$
	.50	.30	.15	.05

   (b) $E(X) = 0(.5) + 1(.30) + 2(.15) + 3(.05) = \mathbf{.75}$

3. (a)

$$\begin{aligned}P(5\text{ heads}) &= P(\text{roll }5)P(5\text{ heads with }5\text{ coins}) + P(\text{roll }6)P(5\text{ heads with }6\text{ coins})\\ &= (1/6)\cdot(1/2)^5 + (1/6)\cdot 6(1/2)^5(1/2)\\ &= \frac{1}{48} = \mathbf{.0208}\end{aligned}$$

   (b)

$$\begin{aligned}P(\text{roll }5 \mid 5\text{ heads}) &= \frac{P(\text{roll }5)P(5\text{ heads} \mid \text{roll }5)}{P(5\text{ heads})}\\ &= \frac{(1/6)\cdot(1/2)^5}{P(5\text{ heads})}\\ &= \frac{1}{4} = \mathbf{.25} \text{ (approx.)}\end{aligned}$$

4. Let A be the event of that a car has an alarm system installed, and let S be the event that a car is stolen.

$$\begin{aligned}P(A|S) = \frac{P(A)P(S|A)}{P(S)} &= \frac{P(A)P(S|A)}{P(A)P(S|A) + P(A')P(S|A')}\\ &= \frac{(.10)(.01)}{(.10)(.01) + (.90)(.02)} = \mathbf{.0526} \text{ (approx.)}\end{aligned}$$

5. (a) $3!(.20)(.50)(.30) = \mathbf{.18}$

   (b) P(at least one pays by cash) = 1 − P(none pay by cash) = $1 - (.80)^3 = \mathbf{.488}$

   (c)

$$\begin{aligned}\text{P(all by cash|at least one by cash)} &= \frac{\text{P(all by cash} \cap \text{at least one by cash)}}{\text{P(at least one by cash)}}\\ &= \frac{\text{P(all by cash)}}{\text{P(at least one by cash)}} = \frac{.20^3}{1 - .80^3}\\ &= \mathbf{.0164} \text{ (approx.)}\end{aligned}$$

6. (a) $\lambda = 2$. Probability is $1 - F(3; 2) = \mathbf{0.143}$ (approx.)
   (b) $\lambda = 2 \times 5 = 10$. Probability is $\dfrac{e^{-10}10^{15}}{15!} = \mathbf{0.035}$ (approx.)

7. (a) Let X = number (from the 25) who buy a large coffee. $X \sim \text{Bin}(n = 25, p = .60)$.
   The mean is $\mu_X = np = 25(.60) = \mathbf{15}$ and the standard deviation is $\sigma_X = \sqrt{np(1-p)} = \sqrt{25(.60)(.40)} = \mathbf{2.45}$ (approx.).
   (b) $P(X > \mu + \sigma) = P(X \geq 18) = 1 - B(17; 25, .60) = 1 - .846 = \mathbf{.154}$
   (c) The revenue in dollars is given by $R = (2.00)X + (1.50)(25 - X) = .50X + 37.5$. And so the expected revenue is $E(X) = E(.50X + 37.5) = .50E(X) + 37.5 = \mathbf{45}$ dollars.

8. X = number of times six appears on 300 rolls of a die. $X \sim \text{Bin}(n = 300, p = 1/6)$, $\mu_X = np = 50$, $\sigma_X = \sqrt{np(1-p)} = 6.455$ (approx.)

$$\begin{aligned} P(X \geq 60) &= 1 - P(X \leq 59) \\ &= 1 - \Phi\left(\frac{59.5 - \mu_X}{\sigma_X}\right) \\ &= 1 - \Phi(1.47) = 1 - .9292 \\ &= \mathbf{.0708} \end{aligned}$$

9. (a)

$$\begin{aligned} P(39 < X < 42) &= \Phi\left(\frac{42 - \mu}{\sigma}\right) - \Phi\left(\frac{39 - \mu}{\sigma}\right) \\ &= \Phi(1.33) - \Phi(-0.67) = .9082 - .2514 \\ &= \mathbf{.6568} \end{aligned}$$

   (b) Want to find the value of c that satisfies $P(X > c) = .15$ or $P(X \leq c) = .85$. This is equivalent to

$$.85 = \Phi\left(\frac{c - \mu}{\sigma}\right)$$

   and so

$$\frac{c - \mu}{\sigma} = 1.04 \implies c = \mu + \sigma(1.04).$$

   Therefore $c = \mathbf{41.56}$
   (c)

$$P(\text{at least one}) = 1 - P(\text{none}) = 1 - (1 - .6568)^4 = \mathbf{.9861} \quad \text{(approx.)}$$

10. A type II error occurs when the Null hypothesis is not rejected when it is false.

11. (a) Average of the endpoints = **174.15** cm.
   (b) The width of the interval is .68. Since the sample is large ($n = 100$) the width of the CI can also be expressed as $2z_{\alpha/2}\frac{s}{\sqrt{n}}$. Therefore

$$s = .68\frac{\sqrt{n}}{2z_{\alpha/2}} = .68\frac{\sqrt{100}}{2(1.96)} = \mathbf{1.73} \text{ cm (approx.)}$$

12. Here $\alpha = .10$. Need $\chi^2_{.05,22} = 33.924$, $\chi^2_{.95,22} = 12.338$. 90% CI for SD has endpoints:

$$\frac{\sqrt{(n-1)}s}{\sqrt{\chi^2_{.05,22}}} = 1.53, \qquad \frac{\sqrt{(n-1)}s}{\sqrt{\chi^2_{.95,22}}} = 2.54 \quad \text{(both approx.)}$$

A 90% CI is **(1.53,2.54)**.

13.

$$H_0 : p = .61 \quad \text{vs.} H_a : p \neq .61$$

Test statistic value

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = 2.356 \quad \text{(approx.)}$$

P-value $= 2(1 - \Phi(2.36)) = 2(1 - .9909) = .0182$.
P-value $< .05 \quad \Rightarrow$. Therefore we reject the null hypothesis.

14. (a) $\sum x_i = 19$, $\sum x_i^2 = 267$, $\bar{x} = 2.375$, $s^2 = 31.696$, $s = 5.630$, $\alpha/2 = .025$, $t_{.025,7} = 2.365$

$$\text{upper/lower limit} = 2.375 \pm (2.365)\frac{5.630}{\sqrt{8}} = 7.08, -2.33$$

A 95% CI is **(-2.33,7.08)**.

(b)

$$\text{H}_0 : \mu = 5 \quad \text{vs.} \quad \text{H}_a : \mu < 5$$

Rejection region is $t \leq -t_{.05,7} = -1.895$
Test statistic value is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.375 - 5}{5.63/\sqrt{8}} = -1.318.$$

The test value is not in the rejection region. Thus the **sample does not provide evidence** the average that the average Sit and Reach Test result for men is less than 5 cm. (Alternatively : P-value $> .102 > \alpha$)

15. Let $D = B - A$.

$$H_0 : \mu_D = 2 \quad \text{vs} \quad H_a : \mu_D > 2 \quad \text{(upper-tailed test)}$$

B-A = D	1	4	-2	0	1	-1	0	5

$\bar{x}_D = 1$, $s_D = 2.39046$

$$\text{test value} = t = \frac{1-2}{s_D/\sqrt{8}} = -1.18 \text{ (approx.)}$$

Negative value, can't be in rejection region (its an upper-tailed test) . Data does not suggest blood pressure is reduced by more than 2 units.

16.

$$H_0 : \mu_W - \mu_M = 0 \quad \text{vs} \quad H_a : \mu_W - \mu_M > 0 \quad \text{(upper-tailed test)}$$

The rejection region is $z \geq z_{.01} = 2.33$. Test statistic value

$$z = \frac{(\bar{x}_W - \bar{x}_M) - 0}{\sqrt{\frac{s_W^2}{n_W} + \frac{s_M^2}{n_M}}} = 2.32$$

SInce z(test) $= 2.32 < 2.33$ we fail to reject the null hypothesis. The data does not suggest that women's mean body temperature is higher then men's.

17.

$$H_0 : p_w = .25, p_{sp} = .25, p_{su} = .25, p_f = .25$$
$$H_a : \text{one of these proportions is not } .25$$

	Winter	Spring	Summer	Fall
Observed	338	345	382	335
Expected	350	350	350	350

$$\chi^2 = \sum_{season} \frac{(observed - expected)^2}{expected} = 4.05$$

From the Table we see that the $.10 < P - value < .90$. Therefore $P - value > \alpha$, and thus we reject the null hupothesis; the data does not provide enough evidence for the hypothesis of equal proportions.