(Marks)

(15) 1. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist and use $-\infty$ or ∞ as appropriate. Show your work.

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-5x+6}{x^2-9}$

(b) $\displaystyle\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3}$

(c) $\displaystyle\lim_{x\to -2^+} \frac{x-2}{x+2}$

(d) $\displaystyle\lim_{x\to -2} \frac{\frac{4}{x+6}-1}{x+2}$

(e) $\displaystyle\lim_{x\to -\infty} \frac{4x^4+3x^2+2}{5x^3-2x+7}$

(f) $\displaystyle\lim_{x\to 4^-} \frac{2\,|x-4|}{x-4}$

(4) 2. Use the graph of the function $f(x)$ below to find the following. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\displaystyle\lim_{x\to -\infty} f(x)$ = ________________

(b) $\displaystyle\lim_{x\to -1} f(x)$ = ________________

(c) $\displaystyle\lim_{x\to 2^-} f(x)$ = ________________

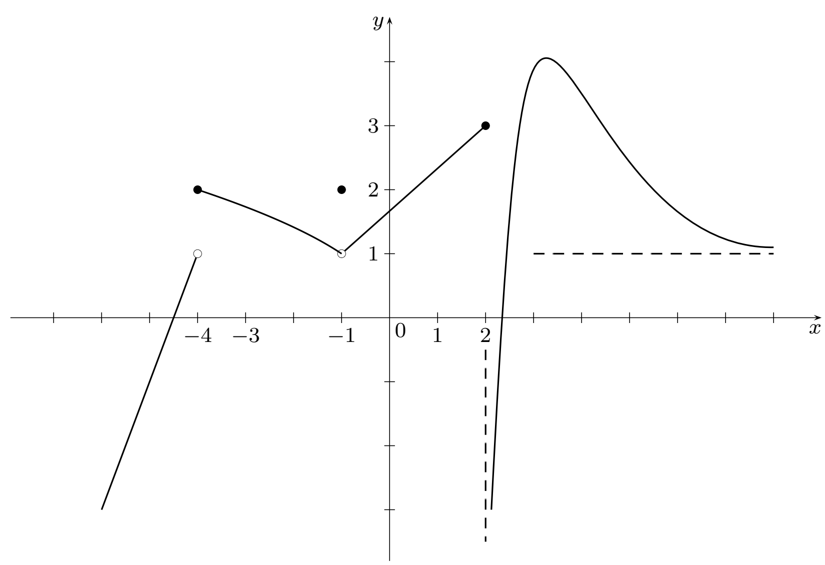
(d) $\displaystyle\lim_{x\to 2^+} f(x)$ = ________________

(e) $\displaystyle\lim_{x\to +\infty} f(x)$ = ________________

(f) $\displaystyle\lim_{x\to -4} f(x)$ = ________________

(g) $f(-1)$ = ________________

(h) $f(-4)$ = ________________



(3) 3. Find the point(s) of discontinuity of the function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \frac{x+3}{(x-5)(x+2)} & \text{if } x < 1 \\ \frac{-2}{x+5} & \text{if } x \geq 1 \end{cases}$$

(3) 4. Find the value(s) of the constant k such that the following function $f(x)$ is continuous for all real numbers.

$$f(x) = \begin{cases} x^2 + k^2 x & \text{if } x \leq 1 \\ 5k + 7x & \text{if } x > 1 \end{cases}$$

(Marks)

(5) 5. (a) State the limit definition for the derivative of a function $f(x)$.

(b) Use the above definition to find the derivative of $f(x) = \dfrac{1}{2-3x}$.

(c) Use derivative rules to check your answer from (b).

(5) 6. Given $x^2y^2 = (x+y)^2 - 5$

(a) Find y'

(b) Find the equation of the tangent line to the curve at the point $(1, 2)$.

(27) 7. Find the derivative for each of the following functions. **Do not simplify your answers.**

(a) $y = 7x^2 - \sqrt[3]{x} + 2x^e + \dfrac{2}{\sqrt{x}} + e^\pi$

(b) $y = e^{3-4x}\ \csc(5x)$

(c) $y = \ln\left(\frac{x^5\cdot(2x-1)^4}{\tan^6(x)}\right)$

(d) $y = \frac{x^2+1}{x^3+x-1}$

(e) $y = \left(e^x + \sin(x^2)\right)^4$

(f) $y = \frac{1+\cot(2x)}{1-\ln(x)}$

(g) $y = 5^x \cos(x^5)$

(h) $y = (x+1)^{x^2}$

(10) 8. Given $f(x) = \dfrac{3x^2}{x^2+3}$ with $f'(x) = \dfrac{18x}{(x^2+3)^2}$ and $f''(x) = \dfrac{54(1-x^2)}{(x^2+3)^3}$.

(a) List, if any, x and y intercepts, vertical and horizontal asymptotes, intervals where $f(x)$ is increasing and decreasing, relative extrema, intervals where $f(x)$ is concave up and concave down, points of inflection.

(b) Sketch a labelled graph of $f(x)$.

(4) 9. Use the **second derivative test** to find all relative (local) extrema of $f(x) = x^4 - 18x^2 + 5$.

(4) 10. Find the absolute (global) extrema of $f(x) = x^3 + 3x^2 - 9x + 2$ on the interval $[-2, 2]$.

(Marks)

(5) 11. A tennis club has a membership of 708 people, each of whom is paying an annual fee of \$530. The club has determined that for each \$10 increase in fees there is a drop in membership of 12 people. What should be the fee to have a maximum income? (Be sure to use a test to confirm that this is a maximum.)

(5) 12. The owner of Rancho Abbott has 3000 meters of fencing material to enclose a rectangular piece of grazing land along the straight portion of a river. If fencing is not required along the river, what are the dimensions of the largest area he can enclose? (Be sure to use a test to confirm that this is a maximum.)

(5) 13. The demand function for a product is given by $p = \sqrt{81 - x}$ for $0 \leq x \leq 81$.

(a) Find the price elasticity of demand, η, when $x = 65$.

(b) Is the demand elastic or inelastic when $x = 65$? Interpret your answer.

(c) Find the value of x such that the demand is unit elastic. Interpret your answer.

(5) 14. Jack and Jill run a small business from the basement of their home, packing and distributing cases of homemade cookies. The cost function in dollars is $C(x) = \frac{1}{3}x^3 + 60x^2 + 500x$ and the demand function in dollars per unit is $p = \frac{2}{3}x^2 + 15x + 2500$.

(a) What is the marginal cost function?

(b) What is the revenue function?

(c) What is the profit function?

(d) If they can produce at most 54 cases of cookies, how many cases of cookies should be produced for maximum profit? (Be sure to confirm that this is a maximum.)

(Marks)

Answers

1. (a)$1/6$ (b)6 (c)$-\infty$ (d)$-1/4$ (e)$-\infty$ (f)-2
2. (a)$-\infty$ (b)1 (c)3 (d)$-\infty$ (e)1 (f)D.N.E. (g)2 (h)2
3. $x = -2$ (be sure to justify) 4. $k = -1, 6$

5. (a)$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ (b)$f'(x) = \dfrac{3}{(2-3x)^2}$ (c) use quotient or chain rules

6. (a)$y' = \frac{2(x+y)-2xy^2}{2x^2y-2(x+y)}$ (b) $y = x+1$

7. (a)$y' = 14x - \frac{1}{3}x^{-2/3} + 2e\,x^{e-1} - x^{-3/2}$ (b)$y' = e^{3-4x}(-4)\csc(5x) + e^{3-4x}\big(-\csc(5x)\cot(5x)5\big)$

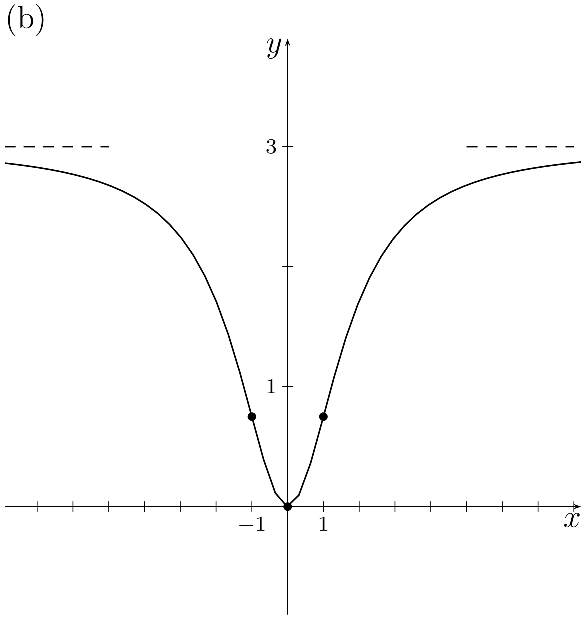
(c)$y' = \frac{5}{x} + \frac{8}{2x-1} - \frac{6\sec^2(x)}{\tan(x)}$ (d)$y' = \frac{2x(x^3+x-1)-(3x^2+1)(x^2+1)}{(x^3+x-1)^2}$

(e)$y' = 4\Big(e^x + \sin(x^2)\Big)^3\Big(e^x + 2x\cos(x^2)\Big)$

(f)$y' = \frac{-\csc^2(2x)2(1-\ln(x))-(1+\cot(2x))(-\frac{1}{x})}{(1-\ln(x))^2}$ (g)$y' = 5^x\ln(5)\cos(x^5) + 5^x\big(-\sin(x^5)5x^4\big)$

(h)$y' = (x+1)^{x^2}\left(2x\ln(x+1) + \dfrac{x^2}{x+1}\right)$

8. (a) x-int:(0,0) y-int:(0,0) VA: none HA:$y=3$ Dec:$(-\infty,0)$ Inc:$(0,\infty)$ Rel. Min:(0,0)
CU:$(-1,1)$ CD:$(-\infty,-1)\cup(1,\infty)$ IP:(1,0.75) ; (−1,0.75)
(b)



9. Rel. Max.:(0,5)
Rel. Min.:$(3,-76)$ and $(-3,-76)$
10. Abs. Min.:$(1,-3)$ Abs. Max.:$(-2,24)$
11. \$560
12. dimensions are 750 by 1500 meters
13. (a)$\eta = \frac{2(x-81)}{x}$; $\eta(65) = -0.49$
(b)since $|\eta(65)| = |-0.49| < 1$ inelastic
(c)$x = 54$ for unit elasticity
14 (a)$C'(x) = x^2 + 120x + 500$
(b)$R(x) = \frac{2}{3}x^3 + 15x^2 + 2500x$
(c)$P(x) = \frac{1}{3}x^3 - 45x^2 + 2000x$
(d) 40 cases of cookies