(Marks)

(14) 1. Evaluate the following limits.
Identify the limits that do not exist, and use ∞ or $-\infty$ as appropriate.
Show your work.

(a) $\displaystyle\lim_{x\to -1} \frac{2x^2 - x - 3}{x^2 - 3x - 4}$

(b) $\displaystyle\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

(c) $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

(d) $\displaystyle\lim_{x\to \infty} \frac{4x^3 + 3x^2 - 7x - 3}{14 - x - 3x^2}$

(e) $\displaystyle\lim_{x\to 0} \frac{\tan x \, \cos x}{x}$

(f) $\displaystyle\lim_{x\to -2^-} \frac{2}{x+2}$

(g) $\displaystyle\lim_{x\to 3^-} \frac{2|x-3|}{x-3}$

(4) 2. Use the graph of the function $y = f(x)$ to find the following. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\displaystyle\lim_{x\to\infty} f(x) =$ ______________

(b) $\displaystyle\lim_{x\to -1} f(x) =$ ________________

(c) $\displaystyle\lim_{x\to 2^-} f(x) =$ _______________

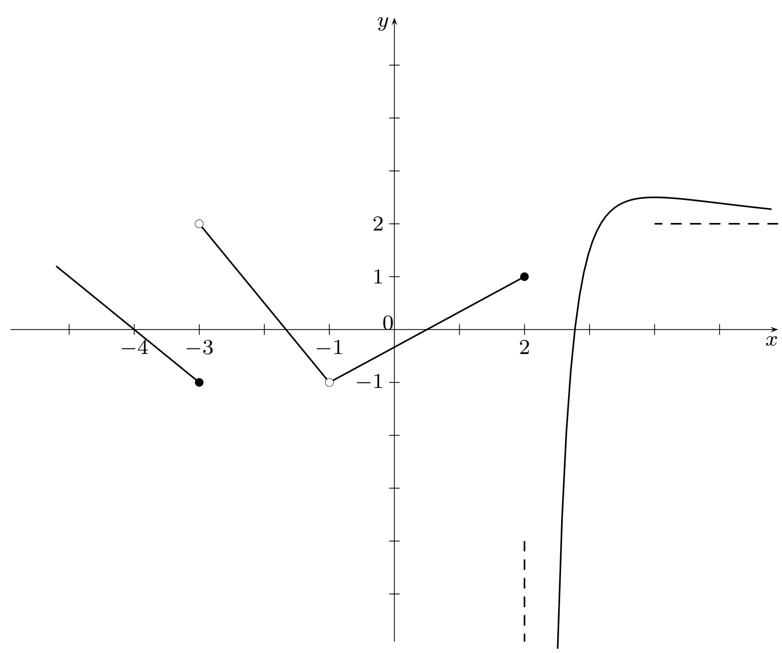
(d) $\displaystyle\lim_{x\to 2^+} f(x) =$ _______________

(e) $\displaystyle\lim_{x\to -\infty} f(x) =$ _________________

(f) $\displaystyle\lim_{x\to -3} f(x) =$ _________________

(g) $f(-3) =$ __________________

(h) $f(-1) =$ ____________________



(3) 3. Find the points(s) of discontinuity of this function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{x-2}{(x-3)(x+1)} & \text{for} \quad x < 4 \\[2ex] \dfrac{3}{x-5} & \text{for} \quad x \geq 4 \end{cases}$$

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(3) 4. Find the value(s) of the constant **k** such that the following function $f(x)$ is continuous for all real numbers.

$$f(x) = \begin{cases} -x^2 - 7k & \text{for} \quad x < 3 \\ k^2 - \dfrac{81}{x} & \text{for} \quad x \geq 3 \end{cases}$$

(6) 5. (a) Use the limit definition of the derivative to find the derivative of $f(x)$ if $f(x) = 2x^2 - 7x - 4$.

(b) Check your answer using the derivative rules.

(c) Find the equation of the tangent line to $f(x)$ in part (a) at the point $(1, -9)$.

(28) 6. Find $\dfrac{dy}{dx}$ for each of the following. **Do not simplify your answer**.

(a) $y = e^{x^4 - x} - \sqrt{x} + \dfrac{2}{x^3} - x^{\ln(3)} + \log_3 (5x^4 - 2x)$

(b) $y = 5^{3x} \csc^4 (3x^4)$

(c) $y = (4x^5 + 3x)^3 \, (3x^2 - 1)^5$

(d) $y = (3x^4 - 2)^{x^3 + 1}$

(e) $y = \ln \left(\dfrac{x^5 \, . \, (3x - 4)^3}{\sqrt[5]{7x - 3} \, \cot^3(x)} \right)$ hint: use logarithmic rules

(f) $5y - 3x^4y^3 + 6x^4 - 4y = 3 + 5x^6y^3$

(g) $y = \dfrac{3x + \cos(x)}{1 - \ln(x)}$

(4) 7. Determine the $x-$value(s) where $f(x)$ has a horizontal tangent(s) given $f(x) = \dfrac{x^2}{x - 1}$

(4) 8. Given the function $f(x) = \sqrt{3 - 2x}$, determine $f'''\left(-\frac{1}{2}\right)$.

(4) 9. Use the second derivative test to determine the relative extrema of $f(x) = -x^3 - 6x^2 - 9x - 2$

(4) 10. Find the absolute (global) extrema of $f(x) = 2x^3 - x^2 + 2$ on the interval $\left[-\frac{1}{2}, 2\right]$.

(10) 11. Given $f(x) = \dfrac{2x^2 - 8}{x^2 - 16}$; $f'(x) = \dfrac{-48x}{(x^2 - 16)^2}$; $f''(x) = \dfrac{48(16 + 3x^2)}{(x^2 - 16)^3}$

(a) Find the $y-$intercept, $x-$intercept, any vertical and horizontal asymptotes, relative extrema and points of inflection (if any).
Find the intervals where f is increasing, decreasing, concave up and concave down.

(b) Sketch a graph of $f(x)$.

(Marks)

(6) 12. A company manufactures protective carrying cases for cellular phones in the form of a rectangular prism. The case is constructed such that its length is 8cm and its volume is 72cm^3. The material for the top and bottom cost $\$0.02/\text{cm}^2$, while material for the sides cost $\$0.08/\text{cm}^2$.

(a) Find the dimensions of the cheapest protective carrying case?

(b) What is the cost of such a case?

(5) 13. The monthly price demand function for a product sold by a monopoly is $p = 8000 - x$ and its average cost is $\overline{C} = 4000 + 4x$.

(a) Determine the quantity that will maximize profit.

(b) Determine the selling price at this optimal quantity.

(c) Determine the revenue at this quantity.

(5) 14. The demand for a product is given by $p = 10\sqrt{100 - x}$ for $0 \leq x \leq 100$.

(a) Find the point at which demand is of unitary elasticity.

(b) What is the price per unit at unitary elasticity?

(c) Find the intervals on which the demand is inelastic and elastic.

(Marks)

Answers

1. a) 1 b) $\frac{1}{2\sqrt{x}}$ c) $-\frac{1}{16}$ d) $-\infty$ e) 1 f) $-\infty$ g) -2

2. a) 2 b) -1 c) 1 d) $-\infty$ e) $+\infty$ f) $D.N.E.$ g) -1 h) $D.N.E.$

3. $x=-1$ or $x=3$ or $x=4$ or $x=5$ 4. $k=-9$; $k=2$

5. a) Use $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ b) $f'(x)=4x-7$ c) $y=-3x-6$

6. a) $\frac{dy}{dx}=e^{x^4-x}(4x^3-1)-\frac{1}{2\sqrt{x}}+\frac{-6}{x^4}-\ln(3)\,x^{\ln(3)-1}+\frac{20x^3-2}{(5x^4-2x)\ln(3)}$

b) $\frac{dy}{dx}=5^{3x}\ln(5)\,(3)\,\csc^4(3x^4)-4(12x^3)\csc(3x^4)\cot(3x^4)\,.\,5^{3x}$

c) $\frac{dy}{dx}=3(20x^4+3)\,(4x^5+3x)^2\,(3x^2-1)^5+5(6x)\,(3x^2-1)^4\,(4x^5+3x)^3$

d) $\frac{dy}{dx}=(3x^4-2)^{x^3+1}\left[3x^2\,\ln(3x^4-2)+\frac{12x^3}{3x^4-2}(x^3+1)\right]$

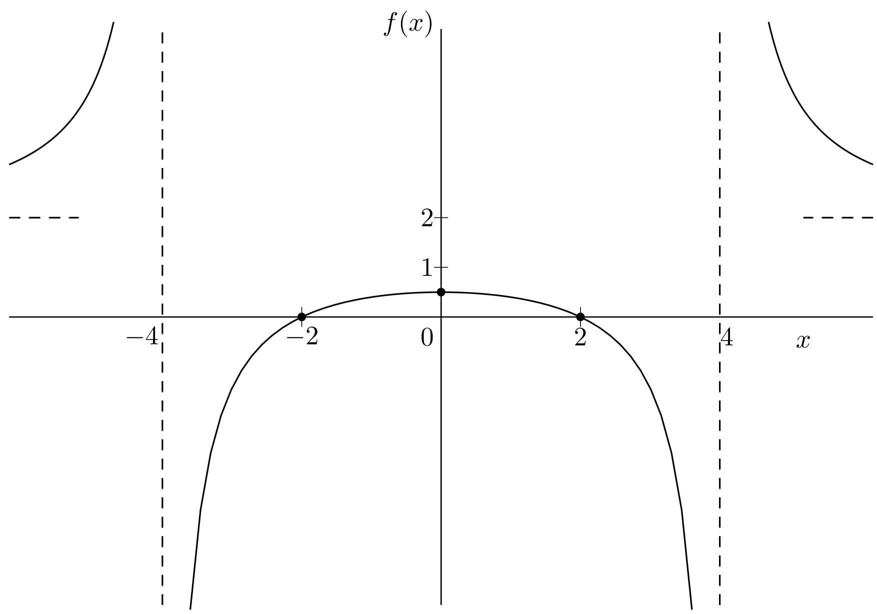
e) $\frac{dy}{dx}=\frac{5}{x}+\frac{9}{3x-4}-\frac{7}{5(7x-3)}-\frac{-3\csc^2(x)}{\cot(x)}$ f) $\frac{dy}{dx}=\frac{30x^5y^3+12x^3y^3-24x^3}{1-15x^6y^2-9x^4y^2}$

g) $\frac{dy}{dx}=\frac{(3-\sin x)(1-\ln x)-\left(\frac{-1}{x}\right)(3x+\cos x)}{(1-\ln x)^2}$ 7. $x=0$ or $x=2$ 8. $-\frac{3}{32}=-0.09375$

9. relative maximum at $(-1,2)$ and relative minimum at $(-3,-2)$

10. absolute maximum is 14 at $x=2$ and absolute minimum is $\frac{3}{2}$ at $x=-\frac{1}{2}$

11 b)



11. a) y-int: $\left(0,\frac{1}{2}\right)$ and
x-int: (2,0) and $(-2,0)$
vertical asymptote:
$x=-4$ and $x=4$
horizontal asymptote: $y=2$
relative maximum: $\left(0,\frac{1}{2}\right)$
PI: none
Inc: $(-\infty,-4)\cup(-4,0)$
Dec: $(0,4)\cup(4,+\infty)$
CU: $(-\infty,-4)\cup(4,+\infty)$
CD: $(-4,4)$

12. a) 8cm by 6cm by 1.5cm ; 12. b) \$5.28
13. a) 400 units ; 13. b) \$7600 per unit ; 13. c) \$3 040 000
14. a) $\frac{200}{3}\approx 66.67$ units ; 14. b) \$57.74 per unit
14. c) inelastic: $66.67\le x\le 100$; elastic: $0\le x\le 66.67$