(Marks)

(28) 1. Find the derivative for each of the following functions. **Do not simplify your answers.**

(a) $y = \dfrac{3x^3 - 3}{\sqrt[3]{x} + x}$

(b) $y = 7^{e^x} + \log_3(e^x) + \ln(x^3) + \sqrt[4]{x^3}$

(c) $y = \ln\left(\dfrac{(5x^3 - 7x)^3\sqrt{x^4 - 3x}}{(x^2 - 6x^5)^2}\right)$ (use log properties)

(d) $y = \dfrac{1 + \sin x}{x + \cos x}$

(e) $y = e^{x + \tan x}$

(f) $y = 2x\sqrt{x + 1}$

(g) $y = (x^2 - 3x^7)^{5x}$

(15) 2. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist and use $-\infty$ or ∞ as appropriate. Show your work.

(a) $\displaystyle\lim_{x \to -2} \frac{x^2 + 6x + 8}{x^2 - 4}$

(b) $\displaystyle\lim_{h \to 0} \frac{\sqrt{2 + h} - \sqrt{2}}{h}$

(c) $\displaystyle\lim_{h \to 0} \frac{\frac{1}{7+h} - \frac{1}{7}}{h}$

(d) $\displaystyle\lim_{x \to \infty} \frac{1 + x^2}{1 - x + 2x^2 - x^3}$

(e) $\displaystyle\lim_{x \to 4^+} \frac{3}{(4 - x)^2}$

(Marks)

(4) 3. Use the graph of the function $f(x)$ below to find the following. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\lim_{x\to\infty} f(x) =$ ________________

(b) $\lim_{x\to 3^+} f(x) =$ ________________

(c) $\lim_{x\to 3^-} f(x) =$ ________________

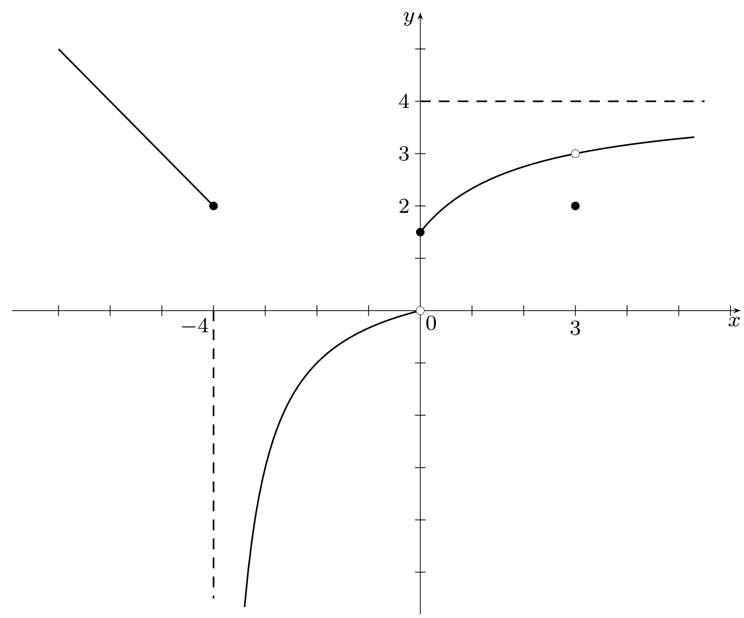
(d) $f(3) =$ ________________

(e) $\lim_{x\to 0} f(x) =$ ________________

(f) $\lim_{x\to -4^+} f(x) =$ ________________

(g) $\lim_{x\to -4^-} f(x) =$ ________________

(h) $\lim_{x\to -\infty} f(x) =$ ________________



(4) 4. Find the point(s) of discontinuity of the function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \frac{4x-1}{x^2-5x-6} & \text{if } x \leq 5 \\ 3x+2 & \text{if } x > 5 \end{cases}$$

(3) 5. Find the value(s) of the constant k such that the following function $f(x)$ is continuous for all real numbers.

$$f(x) = \begin{cases} k^2 - \frac{12k}{x} & \text{if } x \leq -4 \\ 2k - 5x & \text{if } x > -4 \end{cases}$$

(5) 6. (a) State the limit definition for the derivative of a function $f(x)$.

(b) Use the above definition to find the derivative of $f(x) = 2x^2 + 4x$.

(c) Find the point(s) on the function at which the tangent line(s) is (are) horizontal.

(4) 7. Find the absolute (global) extrema of $f(x) = (x^2-4)^3$ on the interval $[-1, 3]$.

(4) 8. Use the **second derivative test** to find all relative (local) extrema of $f(x) = -4x^3 - 6x^2 + 72x + 1$.

(5) 9. Given $e^{2xy} + 2y^3 + 4\sqrt{x} = 5x^2 + e^y$

(a) Find y' .

(b) Find the equation of the tangent line to the curve at the point $(1, 0)$.

(4) 10. Given $y = (3x-1)e^{2x}$; find the third derivative at $x = 0$.

(Marks)

(10) 11. Given $f(x) = \dfrac{2x^2}{9-x^2}$ with $f'(x) = \dfrac{36x}{(9-x^2)^2}$ and $f''(x) = \dfrac{36(3x^2+9)}{(9-x^2)^3}$.

Find, if any:

(a) x and y intercepts,

(b) vertical and horizontal asymptotes,

(c) intervals where $f(x)$ is increasing and decreasing, relative extrema,

(d) intervals where $f(x)$ is concave up and concave down, points of inflection .

(e) Sketch a labelled graph of $f(x)$.

(5) 12. The demand function for a product is given by $p = \sqrt{30-2x}$ for $0 \leq x \leq 15$.

(a) Find the price elasticity of demand, η, when $x = 5$.

(b) Is the demand elastic or inelastic when $x = 5$? Interpret your answer.

(c) Find the value of x such that the demand is unit elastic. Interpret your answer.

(5) 13. Given the cost function $C(x) = 500 + 2x + 0.01x^2$ and the demand function $p(x) = 10 + \dfrac{40}{x}$,

(a) What is the marginal cost function?

(b) What is the revenue function?

(c) What is the profit function?

(d) find the number of units produced in order to have maximum profit.
(Be sure to confirm that this is a maximum.)

(4) 14. A health club charges a \$455 annual fee and has 300 members. The management wants to increase the club's membership by reducing their fees. They estimate that each \$20 reduction would encourage 16 new members. What annual fee will maximize the club's revenue?
(Be sure to use a test to confirm that this is a maximum.)

(Marks)

Answers

1. (a)$y' = \dfrac{9x^2\left(\sqrt[3]{x}+x\right) - (3x^3-3)\left(\frac{1}{3}x^{-2/3}+1\right)}{\left(\sqrt[3]{x}+x\right)^2}$

(b)$y' = 7^{e^x} \,.\, \ln(7) \,.\, e^x + \dfrac{1}{\ln(3)} + \dfrac{3}{x} + \dfrac{3}{4}x^{-1/4}$

(c)$y' = 3\frac{15x^2-7}{5x^3-7x} + \frac{1}{2}\frac{4x^3-3}{x^4-3x} - 2\frac{2x-30x^4}{x^2-6x^5}$ (d)$y' = \frac{\cos x(x+\cos x)-(1-\sin x)(1+\sin x)}{(x+\cos x)^2}$

(e)$y' = e^{x+\tan x} \,.\, (1+\sec^2 x)$ (f)$y' = 2\sqrt{x+1} + \dfrac{2x}{2\sqrt{x+1}}$

(g)$y' = (x^2-3x^7)^{5x}\left(5\ln(x^2-3x^7) + \dfrac{2x-21x^6}{x^2-3x^7}(5x)\right)$

2. (a)$-\frac{1}{2}$ (b)$\frac{1}{2\sqrt{2}}$ (c)$-\frac{1}{49}$ (d)0 (e)∞

3. (a)4 (b)3 (c)3 (d)2 (e)$D.N.E.$ (f)$-\infty$ (g)2 (h)∞

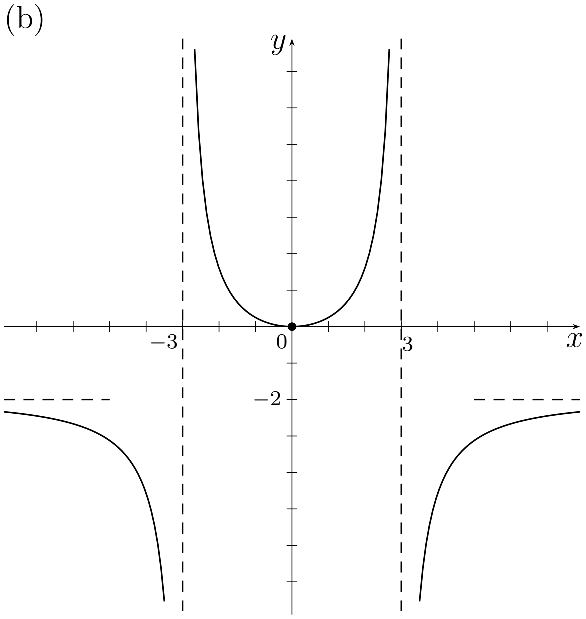
4. $x = -1$; $x = 5$ (be sure to justify) 5. $k = -5, 4$

6. (a)$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ (b)$f'(x) = 4x+4$ (c) $(-1,-2)$

7. Abs. Min.:$(0,-64)$ Abs. Max.:$(3,125)$ 8. Rel. Max.:(2,89) ; Rel. Min.:$(-3,-161)$

9. (a)$y' = \frac{10x-2ye^{2xy}-\frac{2}{\sqrt{x}}}{2xe^{2xy}+6y^2-e^y}$ (b) $y = 8x-8$ 10. 28

11. (a) $x-$int:(0,0) $y-$int:(0,0) VA: $x = -3$; 3 HA:$y = -2$
Dec:$(-\infty,-3)U(-3,0)$ Inc:$(0,3)U(3,+\infty)$ Rel. Min:(0,0)
CU:$(-3,3)$ CD:$(-\infty,-3)\cup(3,\infty)$ IP: none
(b)



12. (a)$\eta = \frac{2(x-15)}{x}$; $\eta(5) = -4$
(b)since $|\eta(5)| = |-4| > 1$ elastic
(c)$x = 10$ for unit elasticity

13. (a)$C'(x) = 2 + 0.02x$
(b)$R(x) = 10x + 40$
(c)$P(x) = -0.01x^2 + 8x - 460$
(d) 400

14. \$415