(9) 1. Name and sketch the following surfaces:

(a) $x^2 = 4(y - z^2)$

(b) $r^2 = z^2 + 4$

(c) $\rho \tan\phi(\cos\theta + \sin\theta) = 2\sec\phi - \rho$

(8) 2. The motion of an object is given by $\mathbf{r}(t) = \langle \ln t^2, \sqrt{8}t, t^2 \rangle$ for $t > 0$.

(a) Find parametric equations of the tangent line to the trajectory at time $t = 1$.

(b) Find an expression for the speed v of the object in terms of t.

(c) Find the curvature κ of the trajectory at time $t = 1$.

(d) Find the tangential and normal components of acceleration at time $t = 1$.

(4) 3. Find the limit if it exists or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(-9x+y)^2}{81x^2+y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2+5y^2}{\sqrt{3x^2+5y^2+1}-1}$

(4) 4. Consider $z = f(x, y) = \sqrt{x^2 + y^2}$.

(a) Find the differential dz.

(b) Use dz to find an approximation for $f(3.06, 3.92)$.

(7) 5. Consider the surface $F(x, y, z) = xz + 2x^2y + y^2z^3 = 11$ and the point $P(2, 1, 1)$.

(a) Find the directional derivative of F at P in the direction of $\mathbf{v} = \langle -1, 1, 1 \rangle$.

(b) Find the maximum rate of increase of F at P?

(c) In what direction (unit vector) does F increase the fastest at P?

(d) Find the equation (in $ax + by + cz = d$ form) of the tangent plane to the surface at P.

(e) Assume that $Q \neq P$ is a point on this tangent plane. What is the directional derivative of F at P in the direction $\overrightarrow{PQ}$?

(f) Find $\dfrac{\partial z}{\partial y}$.

(3) 6. Assume f is a differentiable function and $z = yf(x^2 - y^2)$. Show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$

(5) 7. Find and classify the critical points of $f(x, y) = x^4 + 2y^2 - 4xy$.

(5) 8. Use the method of Lagrange multipliers to find the point on the sphere $x^2 + y^2 + z^2 = 4$ that is farthest from the point $P(1, -1, 1)$.

(12) 9. Evaluate the integrals.

(a) $\displaystyle\int_0^8 \int_{\sqrt[3]{x}}^2 \sin(y^4)\,dydx$

(b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2 + 4)\,dydx$

(c) $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dx\,dy.$

(6) 10. **Set up, but do not evaluate**, triple integrals to find the volume of the region between the sphere $x^2+y^2+z^2=19$ and the upper sheet of the hyperboloid $z^2-x^2-y^2=1$, $z>0$ in

(a) Cartesian coordinates

(b) cylindrical coordinates

(4) 11. Using a suitable change of variables, find the following double integral over T where T is the triangle enclosed by the lines $y-x=0$, $y+x=2$ and the x-axis.

$$\iint_T (x+y)^3\,dx\,dy$$

(2) 12. Let $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{n(x+6)^{3n}}{(3n+1)!}$; evaluate $f^{(27)}(-6)$.

(5) 13. Find the Maclaurin series for the following functions and state the radius of convergence.

(a) $f(x)=\dfrac{x^3}{5+x^2}$

(b) $g(x)=\dfrac{\arctan(3x^2)}{x}$

(5) 14. Approximate $\displaystyle\int_0^{0.1} xe^{-x^3}\,dx$ to six decimal places of accuracy.

(7) 15. Let $f(x)=\dfrac{1}{\sqrt{x}}$

(a) Use the binomial series to expand $f(x)$ as a power series centered at $x=9$ and state the radius of convergence.

(b) If $T_2(x)$ is used to approximate $f(9.5)$, give an upper bound on the error using the Lagrange form of the remainder.

(8) 16. Consider the curve $\mathcal{C}$ having parametric equations: $\begin{cases} x = 2\cos t + 1 \\ y = 3\sin t \end{cases}$ where $t\in\mathbb{R}$.

(a) Find dy/dx and d^2y/dx^2.

(b) Find all the points on $\mathcal{C}$ where the tangent line is vertical or horizontal.

(c) Eliminate the parameter t to express the curve in the form $f(x,y)=d$. Using this equation, identify and sketch $\mathcal{C}$.

(d) **Set up, but do not evaluate,** an integral expression that gives the area bounded by the curve.

(6) 17. Consider the polar curves $r=\cos(3\theta)$ and $r=\dfrac{1}{2}$.

(a) Sketch the two curves on the same axes.

(b) **Set up, but do not evaluate,** an integral expression for the area of the region common to both curves.

(c) **Set up, but do not evaluate,** the integral needed to find the length of $r=\cos(3\theta)$.