1. (6 points) Given the graph of f below evaluate the following expressions. If appropriate use ∞, $-\infty$, or "does not exist".

(a) $\lim_{x\to 2^-} f(x)$

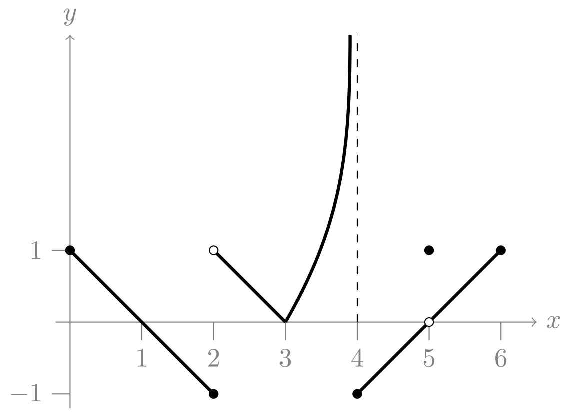
(b) $\lim_{x\to 2^+} f(x)$

(c) $f(2)$

(d) $\lim_{x\to 4^-} f(x)$

(e) $\lim_{x\to 5^-} \dfrac{1}{f(x)}$

(f) $\lim_{x\to \infty} f\left(\dfrac{1}{x}\right)$



2. (10 points) Evaluate the following limits.

(a) $\lim_{x\to 2} \dfrac{3x^2-5x-2}{x^2-4}$

(b) $\lim_{x\to \infty} \left[x-\sqrt{x^2+4x}\right]$

(c) $\lim_{\theta\to 0} \dfrac{\sin^2(5\theta)}{\theta^3-\theta^2}$

(d) $\lim_{x\to -\infty} \dfrac{\sqrt{9x^2+6x+1}}{6x+1}$

(e) $\lim_{x\to 2^-} x\csc(\pi x)$

3. (4 points) Find the derivative of $f(x)=2x^2+x$ using the limit definition of the derivative.

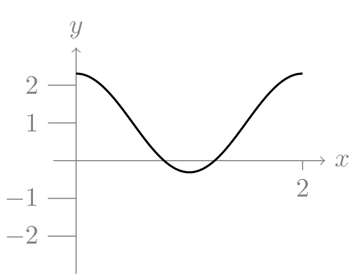
4. (4 points) Let

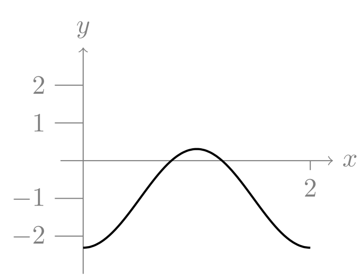
$$f(x)=\begin{cases}\dfrac{|x-3|}{x^2-9} & \text{if } x<3,\\ c & \text{if } x\geq 3.\end{cases}$$

Find all values of c that make the function $f(x)$ continuous at $x=3$.

5. (2 points) Show that $f(x)=3^x-x-3$ has at least one root in $(0,\infty)$.

6. (2 points) Suppose that f is a function which is continuous on the interval $[0,2]$ and differentiable on the interval $(0,2)$. Suppose further that $f(0)=-1$ and $f(2)=1$. One of the following two graphs is the graph of f'. Which one is it? Justify your answer by appealing to a theorem taught in class.





7. (15 points) For each of the following, find $\dfrac{dy}{dx}$.

(a) $y = \dfrac{x^4}{3} + \dfrac{10}{\sqrt[5]{x^2}} + 2^x - \ln 7$

(b) $y = x^7 \ln(x)$

(c) $y = \dfrac{\sin(2x)\sqrt{x^4+5}}{(3x+1)^3}$

(d) $y = (1+x^2)^{\cos x}$

(e) $\ln(x+y) = 1 + \dfrac{1}{x^2}$

8. (5 points) Find the coordinates of all the points on the curve $x^2 + xy + y^2 = 4$ where the tangent line is parallel to the line $y = x + 4$.

9. (3 points) Find the equation of the tangent line to the curve given by $y = \dfrac{3x}{x^2+2}$ at the point with x-coordinate equal to 1.

10. (4 points) Find the absolute extrema of $f(x) = 3(x^2 - 2x)^{2/3}$ on $[1, 4]$.

11. (5 points) A hot air balloon rising straight up from a level field is tracked by a range finder 1500 meters from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at a rate of 0.2 radians per minute. How fast is the balloon rising at that moment?

12. (5 points) The cross section of a tunnel has the form of a rectangle surmounted by a semicircle. The perimeter of this cross section is 18 meters. For what radius of the semi-circle will the cross section have maximum area?

13. (10 points) Given $f(x) = \dfrac{x+1}{(x-3)^2}$, $f'(x) = -\dfrac{(x+5)}{(x-3)^3}$, $f''(x) = \dfrac{2(x+9)}{(x-3)^4}$.

Find (if any):

(a) The domain of f.

(b) The x and y intercept(s).

(c) The vertical and horizontal asymptotes.

(d) Intervals on which f is increasing or decreasing.

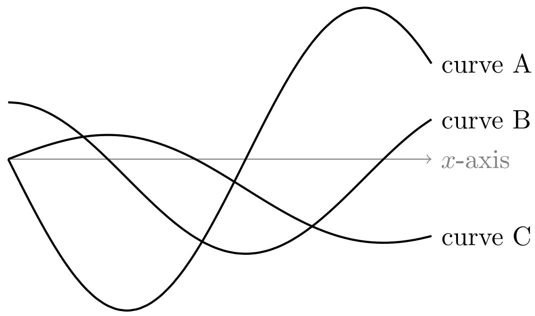
(e) Local (relative) extrema.

(f) Intervals of upward or downward concavity.

(g) Inflection points(s)

(h) On the next page, sketch the graph of f. Label all intercepts, asymptotes, extrema, and points of inflection.

14. (2 points) Let $f(x)$ be some function such that all its higher order derivatives exist. In the picture the graphs of $f(x)$, $f'(x)$, and $f''(x)$ are shown over some interval.



By referring to the picture fill in the blanks below by the letters A, B, and C so that each statement is correct.

- Curve ________ is the graph of $f(x)$.
- Curve ________ is the graph of $f'(x)$.
- Curve ________ is the graph of $f''(x)$.

15. (3 points) Given that $f'(x) = x + 2e^x$ and $f(0) = 5$, find $f(x)$.

16. (4 points) Approximate the integral $\int_0^2 x2^{2x}\,dx$ by a Riemann sum based on partitioning the interval $[0, 2]$ into four equal subintervals.

17. (12 points) Evaluate the following integrals.

(a) $\int \frac{x^3 - 3x + 2}{x^2}\,dx$

(b) $\int \left(e^t + \frac{1}{\sqrt{4t}}\right) dt$

(c) $\int_0^{\pi/6} \sec x\,(\tan x + \cos^2 x)dx$

(d) $\int_{-1}^{3} \big(|x| - 1\big)dx$

18. (2 points) Given $F(x) = \int_0^{x^2} \frac{t}{1+e^t}\,dt$ find $F'(x)$.

19. (2 points) In each part give an example of a function f that fits the description.

(a) f is continuous everywhere and f' has a jump discontinuity.

(b) f is continuous everywhere and f' has an infinite discontinuity.

Answers

1. (a) -1
   (b) 1
   (c) -1
   (d) ∞
   (e) $-\infty$
   (f) 1

2. (a) $\dfrac{7}{4}$
   (b) -2
   (c) -25
   (d) $-\dfrac{1}{2}$
   (e) $-\infty$

3.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{2(x+h)^2+(x+h)-[2x^2+x]}{h} \\ &= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+x+h-[2x^2+x]}{h} \\ &= \lim_{h\to 0} \frac{4xh+2h^2+h}{h} = \lim_{h\to 0} 4x+2h+1 \\ &= 4x+1 \end{aligned}$$

4. $c = -\dfrac{1}{6}$

5. $f(x)$ is continuous on the interval $[1, 2]$, and $f(1) = -1$ while $f(2) = 4$. Since $f(1) < 0 < f(2)$ there is a number c in $[1, 2]$ such that $f(c) = 0$ by the intermediate value theorem. Thus there is a root of $f(x)$ in the interval $(1, 2)$ and hence in $(0, \infty)$.

6. By the mean value theorem there exists a number c in $(0, 2)$ such that

$$f'(c) = \frac{f(2)-f(0)}{2-0} = 1.$$

The graph on the right does not contain a point of the form $(c, 1)$ but the graph on the left does. Thus the graph of f' must be the one on the left.

7. (a) $\dfrac{dy}{dx} = \dfrac{4x^3}{3} - \dfrac{4}{\sqrt[5]{x^7}} + (\ln 2)2^x$
   (b) $\dfrac{dy}{dx} = 7x^6 \ln x + x^6$
   (c) $\dfrac{dy}{dx} = y\left[2\cot(2x) + \dfrac{2x^3}{x^4+5} - \dfrac{9}{3x+1}\right]$
   (d) $\dfrac{dy}{dx} = y\left[\dfrac{2x\cos x}{x^2+1} - \sin x \ln(1+x^2)\right]$
   (e) $\dfrac{dy}{dx} = -1 - \dfrac{2}{x^2} - \dfrac{2y}{x^3}$

8. $(2, -2)$ and $(-2, 2)$

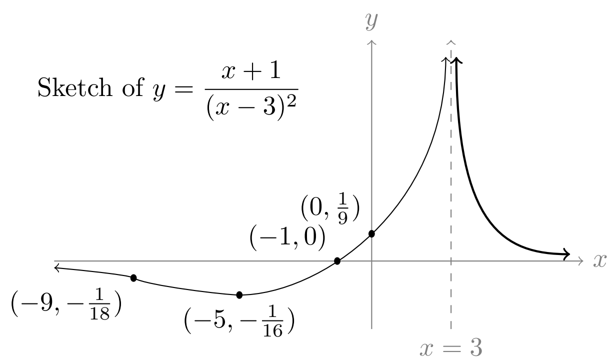
9. $y = \dfrac{1}{3}x + \dfrac{2}{3}$

10. Maximum value is 12, minimum value is 0.

11. 600 meters/minute

12. $r = \dfrac{18}{4+\pi}$

13. (a) $(-\infty, 3) \cup (3, \infty)$
    (b) x-intercept : $(-1, 0)$, y-intercept: $(0, \frac{1}{9})$
    (c) vertical asymptote: $x = 3$, horizontal asymptote: $y = 0$
    (d) f is increasing on $(-5, 3)$, f is decreasing on $(-\infty, -5)$ and $(3, \infty)$
    (e) f has a local minimum value of $f(-5) = -\frac{1}{16}$.
    (f) The graph is concave up over $(-9, 3)$ and $(3, \infty)$, and concave down over $(-\infty, -9)$
    (g) $(-9, -\frac{1}{18})$
    (h)



14. C, B, A

15. $f(x) = \dfrac{x^2}{2} + 2e^x + 3$

16. Using right endpoints: 49/2

17. (a) $\dfrac{x^2}{2} - 3\ln|x| - \dfrac{2}{x} + C$
    (b) $e^t + \sqrt{t} + C$
    (c) $-\dfrac{1}{2} + \dfrac{2}{3}\sqrt{3}$
    (d) 1

18. $F'(x) = \dfrac{2x^3}{1+e^{x^2}}$

19. (a) $f(x) = |x|$ *(Not the only possible example.)*
    (b) $f(x) = x^{1/3}$ *(Not the only possible example.)*