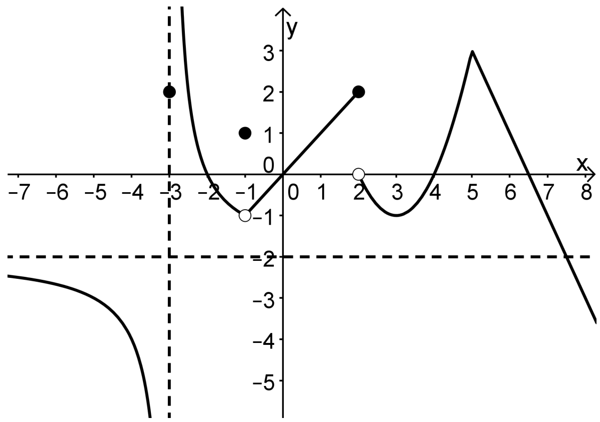
1. (4 points) Given the graph of $y = f(x)$ below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



(a) $\lim\limits_{x \to -\infty} f(x) =$

(b) $\lim\limits_{x \to -3^+} f(x) =$

(c) $\lim\limits_{x \to -1} f(x) =$

(d) $\lim\limits_{x \to 2^+} f(x) =$

(e) $\lim\limits_{x \to 2} f(x) =$

(f) $\lim\limits_{x \to \infty} f(x) =$

(g) List all x-value(s) where f is discontinuous.

(h) List all x-value(s) where f is continuous, but not differentiable.

2. Evaluate the following limits. If a limit doesn't exist, use "does not exist", ∞ or $-\infty$ as appropriate.

(a) (2 points) $\lim\limits_{x \to \infty} \dfrac{(x-1)(7-3x)}{(x^2-1)(2x+5)}$.

(b) (3 points) $\lim\limits_{x \to -3} \dfrac{9-x^2}{3x^2+14x+15}$

(c) (3 points) $\lim\limits_{x \to -4^+} \dfrac{x^2+3x}{x^2-3x-28}$

(d) (3 points) $\lim\limits_{x \to 6^-} \dfrac{|x-6|}{x^2-2x-24}$

(e) (3 points) $\lim\limits_{x \to 4} \dfrac{\dfrac{7}{x+3} - \dfrac{2}{x-2}}{x-4}$

(f) (3 points) $\lim\limits_{x \to 2} \dfrac{3-\sqrt{25-8x}}{2x^2-4x}$

3. (4 points) Find all the x-value(s) where the function $f(x)$ is discontinuous. Justify your answer.

$$f(x) = \begin{cases} \dfrac{5}{x+1} & \text{if } x \leq 0 \\[2ex] \dfrac{-3x^2+5x}{x} & \text{if } 0 < x < 1 \\[2ex] 2x-1 & \text{if } x \geq 1 \end{cases}$$

4. (3 points) Find all the value(s) of the constant k that make the function $g(x)$ continuous for all real numbers. Justify your answer using the definition of continuity.

$$g(x) = \begin{cases} kx + k^2 & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases}$$

5. (6 points) (a) State the limit definition of the derivative:

$f'(x) =$

(b) Use the limit definition of the derivative to find the derivative of $f(x) = \sqrt{2x-4}$.

(c) Check your answer with the differentiation rules.

6. Find the derivative of each of the following functions. You do not need to simplify your answers.

(a) (3 points) $y = (x^2 - 6)^3(1 - 9x)^4$

(b) (4 points) $y = \dfrac{\sqrt[2]{x^5} - 2x + \frac{2}{3x^4}}{3 - \csc(e^x)}$

(c) (3 points) $y = \left(2^{3x+1} + 4x\right)\left(\log_2(4x+1) - 6\right)$

(d) (4 points) $y = (5x + \sqrt{x})^{x^3+1}$

(e) (4 points) $y = \tan^4\left(x^2 + e^{x^2}\right) + 1$

7. (5 points) Use logarithmic differentiation to find y' if $y = \dfrac{e^x(3x-2)^4}{\ln(x)\sin^2(5-x)}$.

8. (6 points) Let $\cos(y - 3x) = x^2y - 2x$

(a) Find $\frac{dy}{dx}$.

(b) Find the slope of the tangent line to the curve at the point $\left(0, \dfrac{\pi}{2}\right)$.

9. (3 points) Find $\dfrac{d^6y}{dx^6}$ if $y = \sin(5x) + e^{-x} + 2x^3$. Justify your answer.

10. (4 points) Find the absolute maximum and minimum values of $f(x) = x^4 - 8x^2 + 10$ on the interval $[-1, 3]$.

11. (5 points) Let$f(x) = \dfrac{(x-3)^3}{x+2}$

(a) Find the critical values of $f(x)$

(b) Find the local extrema of $f(x)$(x-values only).

12. (10 points) Given

$$f(x) = \frac{3(x-1)(x-2)}{x(x-3)}, \qquad f'(x) = \frac{-6(2x-3)}{x^2(x-3)^2}, \qquad f''(x) = \frac{36(x^2-3x+3)}{x^3(x-3)^3}$$

(Note: the numerator of $f''(x)$ does not factor further.)
Determine:

(a) any $x-$ and $y-$intercepts,

(b) any vertical and horizontal asymptotes,

(c) the intervals on which f is increasing and decreasing,

(d) any local maxima and minima of f,

(e) the intervals on which f is concave up and concave down,

(f) any inflection points of f.

(g) Neatly sketch a labeled graph of f, including all information obtained in the previous parts.

13. (5 points) A company wants to fence in a rectangular area of $48,000$ square metres by using the river as one of the sides of the rectangle. The cost of the fence opposite to the river is \$25 per metre, and the cost of the fence on the other two sides is \$15 per metre. Find the dimensions of the rectangle that would minimize the cost. Ensure that your answer is indeed a minimum.

14. (5 points) A farmer knows that when an apple orchard has 100 trees per acre, the average yield will be 630 apples per tree. For each additional tree planted per acre, the average yield per tree is reduced by 5 apples. Find the total number of trees per acre that will maximize the total yield per acre. For this number of trees, what will be the average yield per tree?

15. (5 points) A company sells machines and cannot price it more than \$600. After some marketing research, they determine that the demand curve for these machines is given by $64000x = (p-600)^2$.

(a) Determine the price elasticity of demand function η.

(b) The company has been selling the machines at \$300. Should they raise or lower their price to increase revenue? Justify your answer.

(c) Determine the price at which demand is unit elastic.

Answers: 1)(a)-2 (b)∞ (c)-1 (d)0 (e)DNE (f)$-\infty$ (g)$x=-3,-1,2$ (h)$x=5$

2)(a)0 (b)$\frac{-3}{2}$ (c) $-\infty$ (d) $\frac{-1}{10}$ (e)$\frac{5}{14}$ (f) $\frac{1}{3}$

3) discontinuous at $x=-1,1$

4) $k=-1,2$

5) (a) $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ (b)$\dfrac{1}{\sqrt{2x-4}}$

6) (a)$6x(x^2-6)^2(1-9x)^4 - 36(x^2-6)^3(1-9x)^3$

(b)$\dfrac{\left(\frac{5\sqrt{x^3}}{2} - 2 - \frac{8}{3x^5}\right)(3-\csc(e^x)) - \left(\sqrt{x^5} - 2x + \frac{2}{3x^4}\right)\csc(e^x)\cot(e^x)e^x}{(3-\csc(e^x))^2}$

 Question 15 continues on the next page.

(c) $(3\ln(2)2^{3x+1}+4)(\log_2(4x+1)-6)+(2^{3x+1}+4x)\dfrac{4}{\ln(2)(4x+1)}$

(d) $(5x+\sqrt{x})^{x^3+1}\left(3x^2\ln(5x+\sqrt{x})+\dfrac{(x^3+1)\left(5+\dfrac{1}{2\sqrt{x}}\right)}{5x+\sqrt{x}}\right)$

(e) $4\tan^3\left(x^2+e^{x^2}\right)\sec^2\left(x^2+e^{x^2}\right)\left(2x+2xe^{x^2}\right)$

7) $\dfrac{e^x(3x-2)^4}{\ln(x)\sin^2(5-x)}\left(1+\dfrac{12}{3x-2}-\dfrac{1}{x\ln(x)}+2\cot(5-x)\right)$

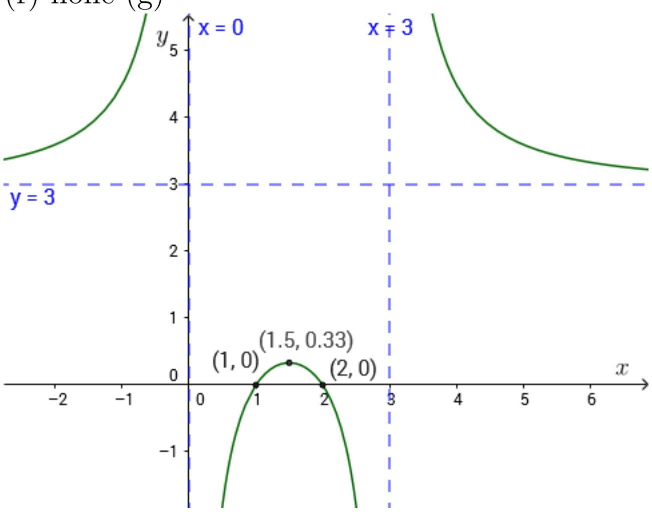
8)(a)$\dfrac{3\sin(y-3x)-2xy+2}{\sin(y-3x)+x^2}$ (b) 5

9)$-5^6\sin(5x)+e^{-x}$

10) absolute minimum: $y=-6$ when $x=2$; absolute maximum: $y=19$ when $x=3$

11) (a)$x=3$, $\frac{-9}{2}$ (b) local minimum at $x=\frac{-9}{2}$ and no local maximum.

12) (a)x-int:$(1,0)$,$(2,0)$; y-int:none (b) VA: $x=0$,$x=3$; HA: $y=3$ (c)increasing: $x\in(-\infty,0)\cup(0,\frac{3}{2})$; decreasing: $x\in(\frac{3}{2},3)\cup(3,\infty)$; (d) local maximum: $(\frac{3}{2},\frac{1}{3})$ (e) CU: $x\in(-\infty,0)\cup(3,\infty)$; CD: $x\in(0,3)$ (f) none (g)



13)240m by 200m

14) 113 trees/acre with 565 apples/tree

15) (a) $\dfrac{2p}{p-600}$ (b) lower the price (c) \$200