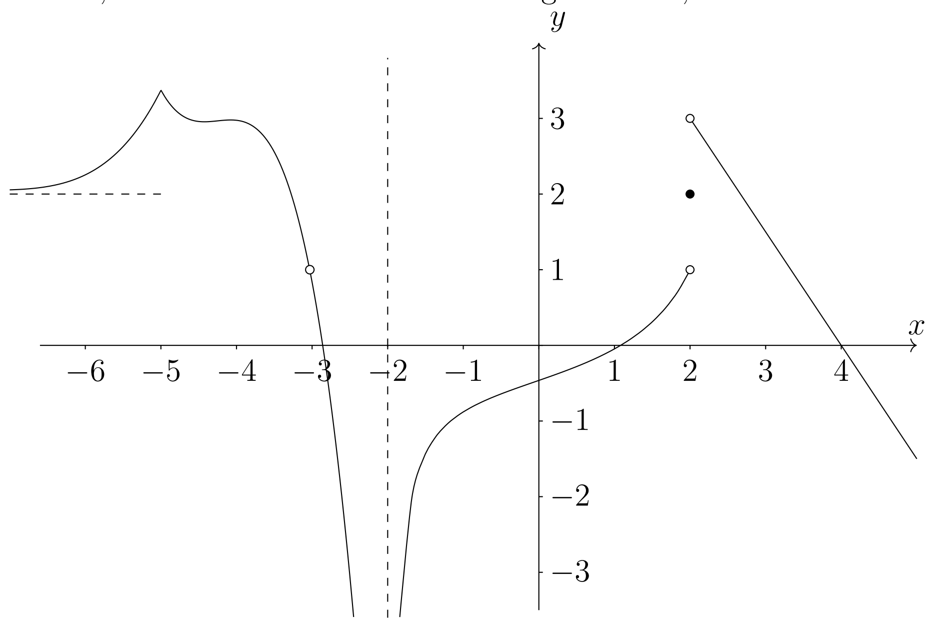
1. (5 points) Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



(a) $\lim_{x\to-3} f(x) =$

(b) $\lim_{x\to2^+} f(x) =$

(c) $\lim_{x\to2} f(x) =$

(d) $\lim_{x\to-2} f(x) =$

(e) $f(2) =$

(f) $\lim_{x\to-\infty} f(x) =$

(g) $\lim_{x\to\infty} f(x) =$

(h) $f'(3) =$

(i) The $x-$values at which $f(x)$ is not continuous:

(j) The $x-$values at which $f(x)$ is not differentiable:

2. (15 points) Evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to3} \frac{2x^2+10x-48}{3x^2-17x+24}$

(b) $\displaystyle\lim_{x\to-5} \frac{5-\sqrt{15-2x}}{x^2+5x}$

(c) $\displaystyle\lim_{x\to1^-} \frac{x^2-4x}{4x-4}$

(d) $\displaystyle\lim_{x\to-2} \frac{x^4-16}{\frac{2}{x}-\frac{x}{2}}$

(e) $\displaystyle\lim_{x\to-\infty} \frac{(x^2+1)(4+7x^3)}{7x^2+10x^3-12x^4}$

3. (3 points) Find the points of discontinuity for the following function $f(x)$. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{30}{(x+5)(x-3)} & \text{if } x<0 \\ -3 & \text{if } x=0 \\ 5x^2+3x-2 & \text{if } x>0 \end{cases}$$

4. (3 points) Find the value(s) of k such that the following function is continuous for **all** real numbers x. Justify using the definition of continuity. [Beware of possible infinite discontinuities]

$$g(x) = \begin{cases} kx & \text{if } x \leq 2 \\ \dfrac{x^2 + 9x + 8}{x + k} & \text{if } x > 2 \end{cases}$$

5. (4 points)

(a) State the limit definition of derivative.

(b) Use this definition to find the derivative of $f(x) = 5x^2 - 3x$.

6. (3 points) Given the function $f(x) = x^3 - 2x$, find an equation for each of the lines tangent to f that has slope 10.

7. (18 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = \sqrt[3]{x} - 3x^{\pi} - \sqrt{x^3} + 4e$

(b) $y = 2x^3 e^{4x} + \tan(5x)$

(c) $y = \dfrac{\sqrt{3x+2}}{x - 7^x}$

(d) $y = \cos^2\Big(\sec(1-x)\Big)$

(e) $e^{x+y^3} = y \ln y - x^2 y$

(f) $y = 3(\sin x + 4x)^x$

8. (4 points) Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

$$y = \frac{(x^3 e^{3x} \sin x)^4}{\ln(2x)}$$

9. (4 points) Find the *slope* of the line tangent to the curve $\dfrac{x}{2y} + 13 = 2x + y^2$ at $(4, -2)$.

10. (3 points) Determine the 275^{th} derivative of the following function.

$$f(x) = -3\cos(5 + 2x)$$

11. (4 points) Given $h(x) = x^2(x-3)^2$, find the absolute extrema for the function $h(x)$ on $[1, 4]$.

12. (4 points) Given $f(x) = 3e^x \sqrt[3]{x}$, find all critical numbers of $f(x)$.

13. (4 points) Use the second derivative test to find all local (relative) extrema.

$$f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + 4$$

14. (10 points) Let $f(x) = \dfrac{(5x+4)(x-4)}{x^2}$. Given $f'(x) = \dfrac{16(x+2)}{x^3}$ and $f''(x) = \dfrac{-32(x+3)}{x^4}$, determine the following characteristics then neatly sketch a graph of $f(x)$ on the following page including all pertinent information.

(a) the domain,

(b) all x- and y-intercepts,

(c) all vertical and horizontal asymptotes,

(d) the intervals on which $f(x)$ is increasing and decreasing,

(e) all local (relative) maxima and minima of $f(x)$,

(f) the intervals on which $f(x)$ is concave up and concave down,

(g) all points of inflection.

15. (5 points) A rectangular storage container with a square base and a total volume of 10m^3 is to be constructed without a cover. The four side walls are cut out of wood panels that cost $\$16/\text{m}^2$ while the square base is made of steel that is currently on sale for $\$40/\text{m}^2$. What should the dimensions of the container be so that the cost is minimized? What is the minimal cost?

16. (5 points) A company which specializes in producing graphics processors wants to maximize their montly profit. At most, they can produce 200 000 processors per month. According to market analysis, their price p per unit should be set to $p = -3x^{\frac{1}{3}} + 240$, where x is the number of units being produced. They have a monthly fixed cost of \$1000, and each processor costs \$40 to make (so the total monthly cost is given by $C = 40x + 1000$). How many processors should be manufactured each month in order to maximize profit? How should the price be set to maximize their monthly profit?

17. (6 points) The demand curve for a product is given by $x = 2400 - 2p^2$ where x is the production level and p is the unit price in dollars.

(a) Determine the price elasticity of demand function η.

(b) What is the price elasticity of demand when the price is \$15.50? Is the demand elastic, inelastic, or unit elastic?

(c) At the price of \$15.50, if the price were increased by 6%, how would the demand be affected?

(d) Determine the price that would maximize the revenue.

ANSWERS

1. (a) 1

(b) 3

(c) D.N.E.

(d) $-\infty$

(e) 2

(f) 2

(g) $-\infty$

(h) -1.5

(i) $x = -3, -2, 2$

(j) $x = -5, -3, -2, 2$

2. (a) 22

(b) $-\dfrac{1}{25}$

(c) ∞

(d) 32

(e) ∞

3. Infinite Discontinuity at $x = -5$. Removable discontinuity at $x = 0$.

4. $k = 3$

5. (a) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(b) $f'(x) = 10x - 3$

6. $f'(x) = 0$ at $x = -2$ and 2
Line 1: $y = 10x - 16$
Line 2: $y = 10x + 16$

7. (a) $\dfrac{dy}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}} - 3\pi x^{\pi-1} - \dfrac{3}{2}x^{\frac{1}{2}}$

(b) $\dfrac{dy}{dx} = 6x^2e^{4x} + 8x^3e^{4x} + 5\sec^2(5x)$

(c) $\dfrac{dy}{dx} = \dfrac{\frac{3}{2}(3x+2)^{-\frac{1}{2}}(x - 7^x) - (1 - 7^x \ln 7)\sqrt{3x+2}}{(x - 7^x)^2}$

(d) $\dfrac{dy}{dx} = 2\cos(\sec(1-x))(\sin(\sec(1-x))\sec(1-x)\tan(1-x)$

(e) $\dfrac{dy}{dx} = \dfrac{e^{x+y^3} + 2xy}{\ln y + 1 - x^2 - 3y^2e^{x+y^3}}$

(f) $\dfrac{dy}{dx} = 3(\sin x + 4x)^x\Big(\ln(\sin x + 4x) + \dfrac{x\cos x + 4x}{\sin x + 4x}\Big)$

8. $\dfrac{dy}{dx} = \dfrac{(x^3e^{3x}\sin x)^4}{\ln(2x)}\Big(\dfrac{12}{x} + 12 + \dfrac{4\cos x}{\sin x} - \dfrac{1}{x\ln(2x)}\Big)$

9. $\dfrac{dy}{dx} = \dfrac{2y-8y^2}{8y^3+2x}$ OR $\dfrac{1-4y}{4x+6y^2-26}$ Slope: $\dfrac{9}{14}$

10. $f^{(275)}(x) = -3\sin(5+2x)2^{275}$

11. Absolute Maximum: 16 (at $x=4$)
Absolute Minimum: 0 (at $x=3$)

12. $x = -\dfrac{1}{3}$ and $x=0$

13. Local Maximum: (0, 4)
Local Minima: (−1, 3.42) and (3, −7.25)

14. (a) Domain: $x \in \mathbb{R}\backslash\{0\}$

(b) $y-$intercept: None; $x-$intercepts: $(-\frac{4}{5}, 0), (4, 0)$

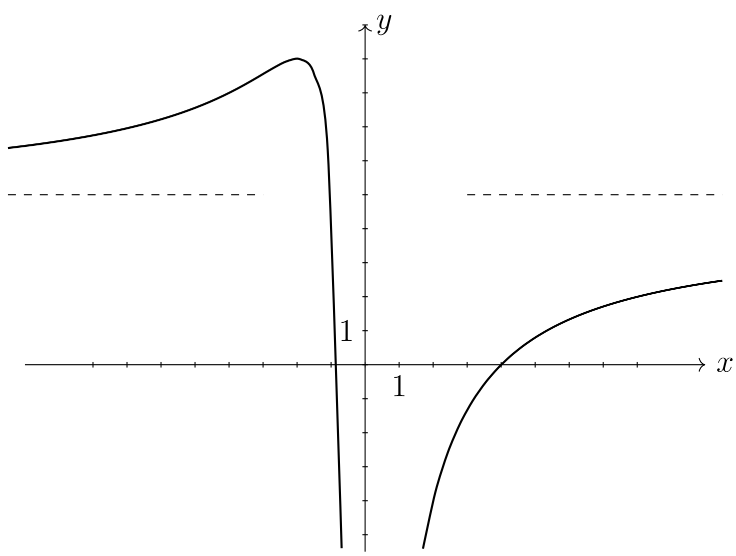
(c) Vertical Asymptote: $x=0$; Horizontal Asymptote: $y=5$

(d) Increasing: $]-\infty,-2[\cup]0,\infty[$; Decreasing: $]-2,0[$

(e) Local Maximum: $(-2,9)$; Local Minimum: None

(f) Concave up: $]-\infty,-3[$; Concave down: $]-3,\infty[\backslash\{0\}$

(g) Inflection Points: $(-3, 8.56)$ //



15. Length: 2m, Width: 2m, Height: 2.5m
Cost= \$480

16. 125,000 processors at \$90 per unit

17. (a) $\eta = \dfrac{-2p^2}{1200-p^2}$

(b) $\eta(15.50) = -0.5$ (Inelastic)

(c) Decrease by 3%

(d) $p = \$20$