1. (a) Yes since it satisfies $A\mathbf{x} = \mathbf{b}$.

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

(d) $\mathbf{a}_4 = 5\mathbf{a}_1 - 2\mathbf{a}_2 + 0\mathbf{a}_3$

2. (a) $a \neq 0$ and $a \neq 3$

(b) $a = 3$

(c) $a = 0$

3. $p(x) = 3 + 2x + 2x^2$

4. $A^{-1} = \begin{bmatrix} 1 & 1 & 10 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 14 \end{bmatrix}$

$(A^T A)^{-1} = \frac{1}{27} \begin{bmatrix} 14 & -1 \\ -1 & 2 \end{bmatrix}$

(b) $AA^T = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 5 \end{bmatrix}$

$\text{rank}(AA^T) = \text{rank}(A^T A) = 2$; or, show that $\det(AA^T) = 0$.

6. $LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$

7. (a) $\begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix} \xrightarrow{-3R2 + R1} \begin{bmatrix} -4 & 0 \\ 3 & 2 \end{bmatrix}$

(b) $E = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

(c) $A = E^{-1}L = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 0 \\ 3 & 2 \end{bmatrix}$

8. (a) Any $\mathbf{u}$ satisfying the following conditions work:
$u_1 = 4 - u_3$, $u_2 = 2$, u_3 free
for example $\mathbf{u} = (3, 2, 1)$

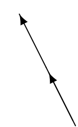
(b) $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(c) $\mathcal{B}_2 = \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(d) T is neither onto nor one-to-one

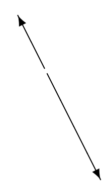
9. (a) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (90° counterclockwise rotation) **or**
$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (90° clockwise rotation)

(b) $B = \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix}$



(c) $T(S(\mathbf{u})) = 2\mathbf{u}$
or for T corresponding to the clockwise rotation

$T(S(\mathbf{u})) = -2\mathbf{u}$



10. $\begin{bmatrix} -A^{-1} & -A^{-1}BA \\ I & 0 \end{bmatrix}$

11. (a) $\det A = -14$

(b) $\det(-2A^{-1}A^T A) = -224$

12. (a) $|A||B||C||A| = |I|$ which implies $|A|^2|B||C| = 1$
So $|A| \neq 0$, $|B| \neq 0$, $|C| \neq 0$

(b) $C^{-1} = A^2 B$

13. (a) $\text{rank}A = 5$ and $\text{nullity}A = 0$

(b) $\text{rank}A = 5$ and $\text{nullity}A = 2$

(c) $\text{rank}A = 1$ and $\text{nullity}A = 1$

14. (a) Yes, for any $A \in H$ and any $k \in \mathbb{R}$, $kA \in H$ since $k^4(abcd) = 0$

(b) No, for example $A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \in H$ and $A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \in H$ yet $A_1 + A_2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \notin H$

15. (a) $\mathcal{B} = \{-1 + x, -2 + x^2\}$

(b) $k = -6$

(c) $p'(x) = 6 \notin V$ yet $p''(x) = 0 \in V$.

16. (a) $(-1, -4, -1)$

(b) $-x + 2y + 3z = 3$

(c) $\cos\theta = \dfrac{13}{14\sqrt{3}}$

17. (a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

(b) Area=3

(c) Volume=13

18. $\text{Proj}_{\mathbf{u}+\mathbf{w}}(\mathbf{u} - 2\mathbf{v}) = \frac{1}{2}(\mathbf{u} + \mathbf{w})$

19. $(AC)\mathbf{b} = \mathbf{b}$ or $A(C\mathbf{b}) = \mathbf{b}$ which implies for every $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{x} = C\mathbf{b}$ is a solution. Therefore, there is a pivot position in every row of A so rank$A = m$.

20. (a) must

(b) cannot

(c) might

(d) must