

1. [3 pts] Below is a list of “algebra rules”. **Some of them are true, some are false.** Circle the true ones and cross the false ones. (Correct answer: 0.5 pts / Incorrect answer: -0.5 pts / No answer: 0 pts.)

•  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

•  $a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$

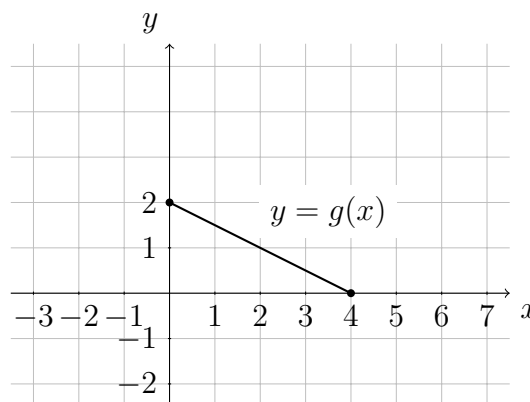
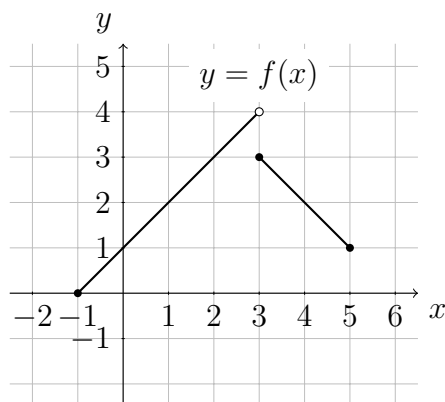
•  $2 \cdot 3^n = 6^n$

•  $(a^m)^n = a^{mn}$

•  $\sqrt{a^2 + b^2} = a + b$

•  $\log_b(x + y) = \log_b x + \log_b y$

2. [6 pts] Let the functions  $f$  and  $g$  be given by the following graphs:



- (a) Evaluate the following:

i.  $f(0)$

ii.  $g(f(4))$

iii.  $g^{-1}(2)$

iv.  $(f/g)(4)$

- (b) Sketch the graph of  $-g(x + 3)$ .

3. [4 pts] Sketch the function  $f(x) = \begin{cases} -2x + 3 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$  and state its range.

4. [5 pts] Let  $A(-1, 5)$  and  $B(3, -3)$  be two points on the plane. Find the:

(a) distance between  $A$  and  $B$ . (Give an exact simplified answer.)

(b) coordinates of the midpoint of the segment between  $A$  and  $B$ .

(c) equation of the line going through the points  $A$  and  $B$ .

(d) equation of the line going through  $A$  and perpendicular to  $y = 2x + 3$ .

(e) equation of the vertical line going through  $A$ .

5. [1 pt] Solve for  $x$ :  $-8x + 5 \leq 7 - 3(x - 1)$ .

6. [2 pts] Simplify leaving only positive exponents:  $\left(\frac{14a^3bc^3}{6a^{-1}(b^2c^{-2})^3}\right)^{-1}$ .

7. [6 pts] Factor completely:

- (a)  $27x^4 - 8x$   
(b)  $5x^3 + x^2 - 45x - 9$   
(c)  $6x^2 - x - 15$
8. [2 pt] Solve  $x^2 - 14x + 50 = 0$  by completing the square. (Simplify as much as possible.)
9. [3 pts] Solve  $-4x^2 + 4x + 7 = 0$  with the quadratic formula. (Simplify as much as possible.)
10. [4 pts] Given  $f(x) = x^2 + 6x + 5$ :
- (a) Find the  $y$ -intercept.  
(b) Find the  $x$ -intercept(s), if any.  
(c) Find the coordinates of the vertex.  
(d) Sketch the graph of  $y = f(x)$ .
11. [1 pt] Use polynomial long division to divide:  $\frac{3x^3 - 2x^2 - 2x + 1}{x^2 - 2}$ .
12. [6 pts] Simplify:
- (a)  $\frac{x^2 - 4}{x^3 + 3x^2} \div \frac{x^2 - 3x + 2}{x^2 + 3x}$   
(b)  $\frac{x}{x-1} + \frac{3}{x-2} + \frac{1}{x^2 - 3x + 2}$   
(c)  $\frac{\frac{4}{x} - 1}{1 - \frac{5}{x-1}}$
13. [15 pts] Solve the following equations:
- (a)  $14 - 9(5x - 1) = 11x$   
(b)  $x^4 + 3x^2 - 10 = 0$   
(c)  $\frac{1}{2} - \frac{2}{x^2 - 1} = \frac{1}{x + 1}$   
(d)  $\log_2(x) + \log_2(x - 2) = 3$   
(e)  $\sqrt{2x - 3} = x - 3$
14. [2 pts] Given  $f(x) = \frac{x + 2}{2x - 5}$ , find a formula for the inverse  $f^{-1}(x)$ .
15. [2 pt] Find the  $x$ -intercept,  $y$ -intercept and asymptotes of  $f(x) = \frac{2x - 3}{6 - x}$  and sketch its graph.
16. [2 pts] Simplify the following radical expressions. For simplicity, you can assume that all variables are positive.

(a)  $\sqrt[3]{48x^4y^2} \sqrt[3]{\frac{x^2}{6y}}$

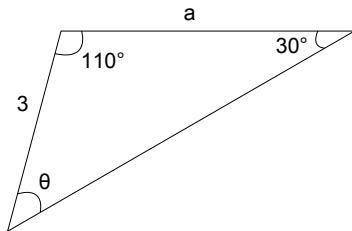
(b)  $(\sqrt{6} + 2\sqrt{3} - 1)(\sqrt{6} + 2\sqrt{3} + 1)$

17. [2 pts] Rationalize the numerator of  $\frac{3 - \sqrt{7-x}}{x+2}$  and simplify.
18. [1 pt] Rationalize the denominator of  $\frac{2}{\sqrt[3]{4x}}$  and simplify.
19. [3 pts] Find the domain of the following functions.
- (a)  $f(x) = \frac{x(x-2)}{(x+3)(x-6)}$
- (b)  $g(x) = \frac{1 - \sqrt{1-x}}{x+2}$
20. [1 pt] Use a calculator to evaluate  $\log_7(2016)$  to 3 decimal places.
21. [3 pts] You invest \$ 2,500 in a “High Interest” savings account that pays an annual interest rate of 0.550% compounded daily. Assuming there are 365 days in each year (no leap years), how much will you have after 7 years? (Give you answer to the nearest cent.)
22. [2 pts] Express  $\log\left(\frac{\sqrt{x^2+3}}{100x^{2016}}\right)$  in terms of the simplest possible logarithms.
23. [2 pts] Given  $3^{2-x} = \frac{27^x}{3}$ , find the exact value of  $x$ .
24. [2 pts] Let  $f(x) = \log_2(x+8)$ . Sketch the graph of  $f$  and indicate all intercepts and asymptotes. State the domain and range of  $f$ .
25. [3 pts] Let  $\theta$  be the angle in standard position with terminal side containing the point  $(-1, 4)$ . Find the exact value of:
- (a)  $\cos \theta$
- (b)  $\csc \theta$
- (c)  $\cot \theta$
26. [2 pts] Find the exact value of two angles  $\theta$  in the interval  $[0, 2\pi)$  with  $\cos \theta = -\frac{1}{2}$ .
27. [2 pts] Prove the identities:
- (a)  $(1 - \cos^2 x)(1 + \cot^2 x) = 1$
- (b)  $\frac{\tan x}{\sec x - 1} = \frac{1 + \cos x}{\sin x}$
28. [3 pts] To measure the height of a statue standing on a base, two sightings 100 meters from the bottom of the base are taken. If the angle of elevation to the bottom of the statue is  $5^\circ$  and the angle of elevation to the top of the statue is  $15^\circ$ , what is the height of the statue?
29. [1 pt] Convert  $\frac{7\pi}{6}$  rad to degrees.
30. [2 pts] Consider the angle  $\theta = -\frac{7\pi}{6}$  rad.
- (a) Sketch  $\theta$  and state the reference angle.
- (b) Without using a calculator find the exact value of  $\cos \theta$ .

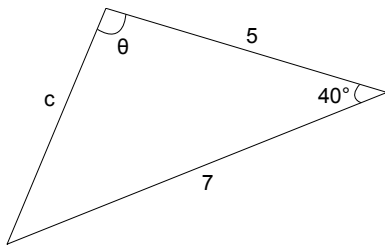
31. [2 pts] Let  $y = -\sin(2\pi x)$ .

- (a) Find the amplitude and period.
- (b) Graph two cycles of this function.

32. [2 pts] For the triangle below, find the angle  $\theta$  and the length  $a$  ( Round your answer to two decimal places).



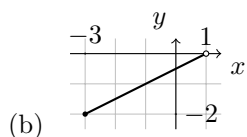
33. [2 pts] For the triangle below, find the angle  $\theta$  and the length  $c$ . ( Round your answers to two decimal places).



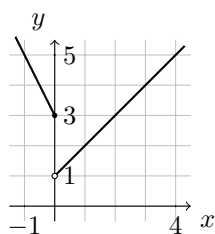
Answers

1.   • TRUE           • TRUE  
     • FALSE          • FALSE  
     • FALSE          • FALSE

2. (a) i. 1  
     ii. 1  
     iii. 0  
     iv. undefined



3. range =  $(1, \infty)$



4. (a)  $4\sqrt{5}$   
     (b)  $(1, 1)$   
     (c)  $y = -2x + 3$   
     (d)  $-\frac{1}{2}x + \frac{9}{2}$   
     (e)  $x = -1$

5.  $x \geq -1$

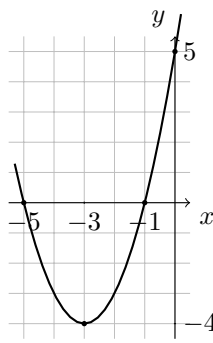
6.  $\frac{3b^5}{7a^4c^9}$

7. (a)  $x(3x - 2)(9x^2 + 6x + 4)$   
     (b)  $(x - 3)(x + 3)(5x + 1)$   
     (c)  $(2x + 3)(3x - 5)$

8. no solution

9.  $\frac{-1 \pm 2\sqrt{2}}{2}$

10. (a)  $(0, 5)$   
     (b)  $(-5, 0), (-1, 0)$   
     (c)  $(-3, -4)$



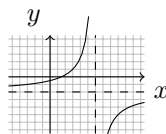
11.  $3x - 2 + \frac{4x - 3}{x^2 - 2}$

12. (a)  $\frac{x + 2}{x(x - 1)}$   
     (b)  $\frac{x + 2}{x - 2}$   
     (c)  $\frac{(4 - x)(x - 1)}{x(x - 6)}$

13. (a)  $x = \frac{23}{56}$   
     (b)  $x = \pm\sqrt{2}$   
     (c)  $x = 3$   
     (d)  $x = 4$   
     (e)  $x = 6$

14.  $f^{-1}(x) = \frac{5x + 2}{2x - 1}$

15.   •  $x$ -int.:  $(3/2, 0)$   
     •  $y$ -int.:  $(0, -1/2)$   
     • hor. asympt.:  $y = -2$   
     • vert. asympt.:  $x = 6$



16. (a)  $2x^2 \sqrt[3]{y}$   
     (b)  $17 + 12\sqrt{2}$

17.  $\frac{1}{3 + \sqrt{7 - x}}$

18.  $\frac{\sqrt[3]{2x^2}}{x}$

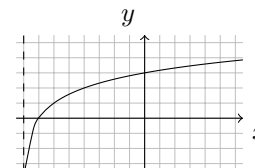
19. (a)  $\mathbb{R} \setminus \{-3, 6\}$   
     (b)  $(-\infty, -2) \cup (-2, 1]$

20. 3.910

21. \$ 2,598.13

22.  $\frac{1}{2} \log(x^2 + 3) - 2 - 2016 \log x$

23.  $x = 3/4$



24.   •  
     • domain =  $(-8, \infty)$   
     • range =  $(-\infty, \infty)$

25. (a)  $-1/\sqrt{17}$   
     (b)  $\sqrt{17}/4$   
     (c)  $-1/4$

26.  $\theta_1 = 2\pi/3, \theta_2 = 4\pi/3$

27. (a) LHS =  
 $(1 - \cos^2 x) \left(1 + \frac{\cos^2 x}{\sin^2 x}\right)$   
 $= (\sin^2 x) \left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right)$   
 $= \sin^2 x \cdot \frac{1}{\sin^2 x} = \text{RHS}$

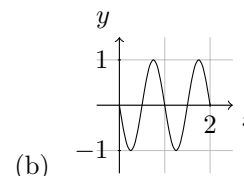
(b) LHS =  $\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} - 1}$   
 $= \frac{\frac{\sin x}{\cos x}}{\frac{1 - \cos x}{\cos x}}$   
 $= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1 - \cos x} = \frac{\sin x}{1 - \cos x}$   
 $= \frac{\sin x \cdot (1 + \cos x)}{(1 - \cos x) \cdot (1 + \cos x)}$   
 $= \frac{\sin x \cdot (1 + \cos x)}{1 - \cos^2 x}$   
 $= \frac{\sin x \cdot (1 + \cos x)}{\sin^2 x} = \text{RHS}$

28. 18.046 m

29.  $210^\circ$

30. (a)  $\theta_R = \pi/6$   
     (b)  $-\sqrt{3}/2$

31. (a)  $A = 1, P = 1$



32.  $\theta = 40^\circ, a \approx 3.856$

33.  $c \approx 4.514, \theta \approx 85.408^\circ$