1. Evaluate the following integrals.

(5) (a) $\displaystyle\int \frac{2x^2+x+3}{x(x+1)^2}\,dx$

(5) (b) $\displaystyle\int \frac{\cot^3(\ln x)\csc^4(\ln x)}{x}\,dx$

(5) (c) $\displaystyle\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

(5) (d) $\displaystyle\int \frac{2x+5}{4x^2+4x+10}\,dx$

(5) (e) $\displaystyle\int_1^6 e^{\sqrt{3x-2}}\,dx$

(5) (f) $\displaystyle\int e^x \arcsin(e^x)\,dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to 0^+} \frac{\arctan\sqrt{x}}{3\sqrt{x}}$

(3) (b) $\displaystyle\lim_{x\to 0^+} (\sin x)^x$

3. Evaluate each improper integral or show it diverges.

(4) (a) $\displaystyle\int_0^\infty \frac{e^{-2x}}{e^{-2x}+3}\,dx$

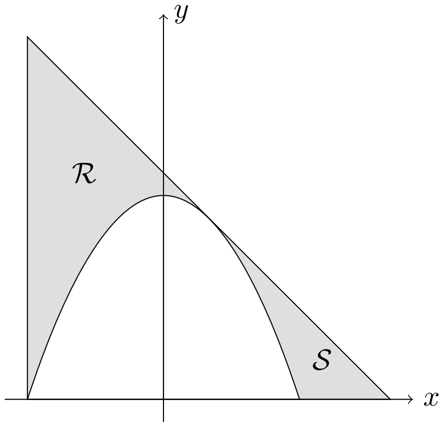
(4) (b) $\displaystyle\int_0^{\pi/3} \tan(2x)\,dx$

(4) **4.** Find the value of $k>0$ such that the area bounded by the parabolas $y=x^2-kx$ and $y=\frac{x^2}{2}$ is equal to 18.

5. Consider the two pictured regions $\mathcal{R}$ and $\mathcal{S}$. The parabola $y=9-\dfrac{x^2}{4}$ and the line $y=-x+10$, which is tangent to this parabola, border these two regions. In each part below, set up, **but do not evaluate**, the volume of the solid of revolution obtained by rotating;

(3) (a) the region $\mathcal{R}$ about the $x-$axis

(3) (b) the region $\mathcal{S}$ about the line $y=-1$



(4) **6.** Solve the following initial value problem.

$$\frac{dy}{dx} = xy\sqrt{1+x^2} \qquad \text{given } y(0) = -1$$

(3) **7.** There are initially 100 bacteria in a petri dish. After two hours, the number of bacteria has tripled. Assuming that the rate of increase of the population of bacteria is proportional to the number of bacteria at the moment, find a formula for the population of bacteria after t hours.

(5) **8.** Find the length of the curve of the function $x = \frac{1}{3}y^{3/2} - y^{1/2}$ from $y = 1$ to $y = 4$.

9. Determine whether the following sequences converge or diverge. In the case of convergence, find the limit.

(2) (a) $\left\{\arcsin\left(\dfrac{3n-2}{5-6n}\right)\right\}$

(2) (b) $\left\{\displaystyle\int_{n+1}^{n+4} \frac{dx}{x+3}\right\}$

10. Suppose that $\{a_n\}$ is a monotonic sequence of positive numbers and that $\sum a_n$ is convergent. Determine whether the following are convergent or divergent. Justify each answer using an appropriate series test.

(1) (a) The sequence $\{a_n\}$

(1) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n - 1}{3 + a_n}$

(2) (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \arctan(a_n)$

(3) **11.** Determine whether the following series converges or diverges. If it converges, find the sum.

$$\sum_{n=0}^{\infty} (\arctan(n+1) - \arctan(n))$$

12. Determine whether the following series converge or diverge. Justify your answers.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n + 3^n}{5^n - 2^n}$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n}$

13. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n+1)}{n^2+3n}$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3n-3)!}{3n \cdot 3^{3n}}$

(3) (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5-n}{2n}\right)^n$

(4) **14.** Find the radius of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdot \ \cdots \ \cdot (2n+1)}{3^n(n+1)!}(x+3)^n$$

(4) **15.** Determine the Taylor series expansion of the function $f(x) = \ln(x+3)$ centered at $x = 2$.

Answers

1. (a) $\ln\left|\dfrac{x^3}{x+1}\right| + \dfrac{4}{x+1} + C$

   (b) $\dfrac{\csc^4(\ln x)}{4} - \dfrac{\csc^6(\ln x)}{6}$+C OR $\dfrac{-\cot^4(\ln x)}{4} - \dfrac{\cot^6(\ln x)}{6}$+C

   (c) $\dfrac{25}{2}\arcsin\left(\dfrac{x}{5}\right) - \dfrac{x\sqrt{25-x^2}}{2}$+C

   (d) $\dfrac{1}{4}\ln\left[\left(x+\dfrac{1}{2}\right)^2 + \dfrac{9}{4}\right] + \dfrac{2}{3}\arctan\left(\dfrac{2x+1}{3}\right)$ +C

   (e) $2e^4$

   (f) $e^x \arcsin(e^x) + \sqrt{1-e^{2x}}$+C

2. (a) $\dfrac{1}{3}$ (b) 1

3. (a) $\dfrac{-1}{2}\ln\left(\dfrac{3}{4}\right)$ (converges) (b) Diverges

4. $k = 3$

5. (a) $V = \displaystyle\int_{-6}^{2} \pi\left[(10-x)^2 - \left(9 - \frac{x^2}{4}\right)^2\right] dx$

   (b) $V = \displaystyle\int_{0}^{8} 2\pi(y+1)\left(10 - y - \sqrt{36-4y}\right) dy$

6. $y = -e^{\frac{1}{3}[(1+x^2)^{3/2}-1]}$

7. $N = 100e^{\ln(\sqrt{3})t} = 100 \cdot 3^{t/2}$

8. $L = \dfrac{10}{3}$

9. (a) $\arcsin(-\frac{1}{2}) = -\dfrac{\pi}{6}$ (converges) (b) $\ln(1) = 0$ (converges)

10. (a) Converges to 0 using Divergence Test

    (b) Diverges by Divergence Test and using part (a)

    (c) Converges by Alternating Series Test

11. Converges to $\frac{\pi}{2}$ (Telescoping series)

12. (a) Converges by Limit Comparison Test with $\sum \left(\frac{3}{5}\right)^n$

    (b) Converges by Limit Comparison Test with $\sum \frac{1}{n^2}$

13. (a) Not absolutely convergent by Limit Comparison Test with $\sum \frac{1}{n}$ however converges by the Alternating Series Test. Answer: Conditionally Convergent

    (b) Diverges by the Ratio Test as $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \to \infty$

    (c) Converges Absolutely by the Root Test as $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \frac{1}{2}$

14. Radius of Convergence $= \frac{3}{2}$ Note: The Interval of Convergence is $\left[\frac{-9}{2}, \frac{-3}{2}\right)$

15. $f(x) = \ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 5^n}(x-2)^n$