1. (10 points) Evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to 2} \frac{3x^2 - 11x + 10}{x^3 - 8}$

(b) $\displaystyle\lim_{x\to 0} \frac{x + \sin(5x)}{\sin(2x)}$

(c) $\displaystyle\lim_{x\to \pi/3^+} \frac{1}{2 - \sec x}$

(d) $\displaystyle\lim_{x\to \infty} \left(x - \sqrt{x^2 + 5x}\right)$

(e) $\displaystyle\lim_{x\to 5^+} \frac{25 - x^2}{|x - 6| - 1}$

2. (3 points) What value of c makes the following function continuous at 2?

$$f(x) = \begin{cases} c^2x + 3c & \text{if } x < 2, \\ x & \text{if } x = 2, \\ cx + c^2 + 2 & \text{if } x > 2. \end{cases}$$

3. (2 points) Give an equation for each horizontal and vertical asymptote of the graph of $f(x) = \dfrac{x + \sin x}{3x + 2}$.

4. (4 points) Use the limit definition of the derivative to find $f'(x)$, where $f(x) = \dfrac{1}{3x + 2}$.

5. (12 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answer.

(a) $y = \dfrac{8}{x} - \sqrt[3]{x} + 2^x$

(b) $y = \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$

(c) $y = \cos^3(6x^2)$

(d) $\ln(x - y) = xy - 2$

6. (4 points) Let $f(x) = \dfrac{2^{x+3}\sqrt{9 - x^2}}{(x + 1)^4(6x + 3)}$. Use logarithmic differentiation to find $f'(0)$. *Simplify your answer.*

7. (3 points) Use the Intermediate Value Theorem to show that the equation $x^3 - 4x + 2 = 0$ has at least one *positive* solution.

8. (3 points) Find all points P on the parabola given by the equation $y = x^2 - 2x$ such that the line joining P and the point $(4, 4)$ is tangent to the parabola.

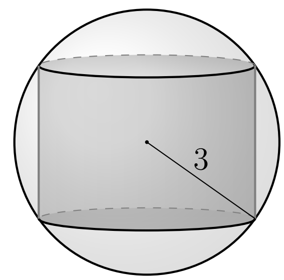
9. (4 points) (a) State the Mean Value Theorem.

(b) Show that if $f(2) = -2$ and $f'(x) \geq 5$ for $x \geq 2$, then $f(4) \geq 8$.

10. (4 points) Consider the curve $\mathcal{C}$ given by the equation $x^3 + y^3 = 8(xy + 1)$. Find an equation for the tangent line to the curve $\mathcal{C}$ at the point $(-1, 1)$.

11. (6 points) A lighthouse is located on a small island 2 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam moving along the shoreline when it is 0.5 km away from P?

12. (6 points) A right circular cylinder is inscribed in a sphere with radius 3. Find the largest possible volume of such a cylinder.



13. (4 points) The position of a particle moving along a straight line at time $t \geq 0$ is given by $s = (t-3)^2 e^{-t}$ where s is measured in meters and t is in seconds.

(a) When is the particle at rest?

(b) When is the particle moving in the positive direction?

14. (4 points) Find the absolute extrema of $f(x) = \dfrac{x+18}{\sqrt{x^2+36}}$ on $[0, 8]$.

15. (10 points) Given

$$f(x) = \frac{8(x^2+4)}{(x+2)^2}, \quad f'(x) = \frac{32(x-2)}{(x+2)^3} \quad \text{and} \quad f''(x) = \frac{(-64)(x-4)}{(x+2)^4}, \text{ find all:}$$

(a) x and y intercepts.

(b) Vertical and horizontal asymptotes.

(c) Intervals of which $f(x)$ is increasing or decreasing.

(d) Local (relative) extrema.

(e) Intervals of upward and downward concavity.

(f) Inflection points.

(g) Find the coordinates of the point(s) where the graph of f intersects its horizontal asymptote.

(h) On the next page, sketch the graph of $f(x)$. Label all intercepts, asymptotes, extrema, and points of inflection.

16. (12 points) Evaluate each of the following integrals.

(a) $\displaystyle\int \left(e^2 - \frac{4}{x} + \sqrt[3]{x^5}\right) dx$

(b) $\displaystyle\int \frac{(x^5+1)^2}{x^4}\, dx$

(c) $\displaystyle\int \sec x(\sec x + \tan x)\, dx$

(d) $\displaystyle\int_0^{\pi/2} \left|\frac{1}{2} - \sin(x)\right| dx$

17. (2 points) Find the derivative with respect to x of $y = \displaystyle\int_0^{x^2} \frac{t}{1+t^2}\, dt$.

18. (5 points) Evaluate $\displaystyle\int_0^2 (2x^2+1)\, dx$ using the definition of the integral as a limit of Riemann sums. You might use the formulas $\displaystyle\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\displaystyle\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, and $\displaystyle\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

19. (2 points) Let f be an even function such that $\displaystyle\int_0^3 f(x)\, dx = 216$ and $\displaystyle\int_{-2}^3 f(x)\, dx = 240$. Evaluate $\displaystyle\int_{-2}^2 (x-2)f(x)\, dx$.

Answers:

1. (a) $\frac{1}{12}$

(b) 3

(c) $-\infty$

(d) $-\frac{5}{2}$

(e) 10

2. $c = -2$

3. One horizontal asymptote; $y = 1/3$, and one vertical asymptote; $x = -2/3$.

4. $f'(x) = \displaystyle\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \cdots = \lim_{h\to 0} \frac{-3}{(3x+3h+2)(3x+2)} = -\frac{3}{(3x+2)^2}$

5. (a) $\dfrac{dy}{dx} = -\dfrac{8}{x^2} - \dfrac{1}{3\sqrt[3]{x^2}} + (\ln 2)2^x$

(b) $\dfrac{dy}{dx} = \dfrac{2x}{(x^2-1)^{1/2}(x^2+1)^{3/2}}$

(c) $\dfrac{dy}{dx} = -36x\cos^2(6x^2)\sin(6x^2)$

(d) $\dfrac{dy}{dx} = \dfrac{1 - xy + y^2}{x^2 - xy + 1}$

6. $f'(0) = 8(\ln 2 - 6)$

7. Let $f(x) = x^3 - 4x + 2$. Note that $f(0) = 2$ and $f(1) = -1$. Since f is continuous on $[0, 1]$ and 0 is between $f(0)$ and $f(1)$, it follows by the Intermediate Value Theorem that there exists a number c in $(0, 1)$ such that $f(c) = 0$. The number c is a positive solution to the given equation.

8. $(2, 0)$ and $(6, 24)$

9. (a) If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

(b) The statement of the problem implies that f is differentiable at all $x \geq 2$, and hence f is continuous at all $x \geq 2$. Therefore, we may apply the Mean Value Theorem with f and the interval $[2, 4]$ to obtain a number c in $(2, 4)$ such that $f'(c) = \dfrac{f(4) - (-2)}{2}$, or equivalently, $f(4) = 2f'(c) - 2$. By the hypothesis $f'(c) \geq 5$, hence $f(4) \geq 2(5) - 2 = 8$.

10. $y = \dfrac{5x + 16}{11}$

11. 17π km/min

12. $12\sqrt{3}\pi$ cubic units

13. (a) $t = 3$ and $t = 5$

(b) $3 < t < 5$

14. The minimum value is $13/5$ and the maximum value is $\sqrt{10}$.

15. (a) No x-intercept, y-intercepts: $(0, 8)$

(b) Vertical asymptote: $x = -2$, horizontal asymptote: $y = 8$

(c) Increasing on $(-\infty, -2)$ and $(2, \infty)$. Decreasing on $(-2, 2)$.

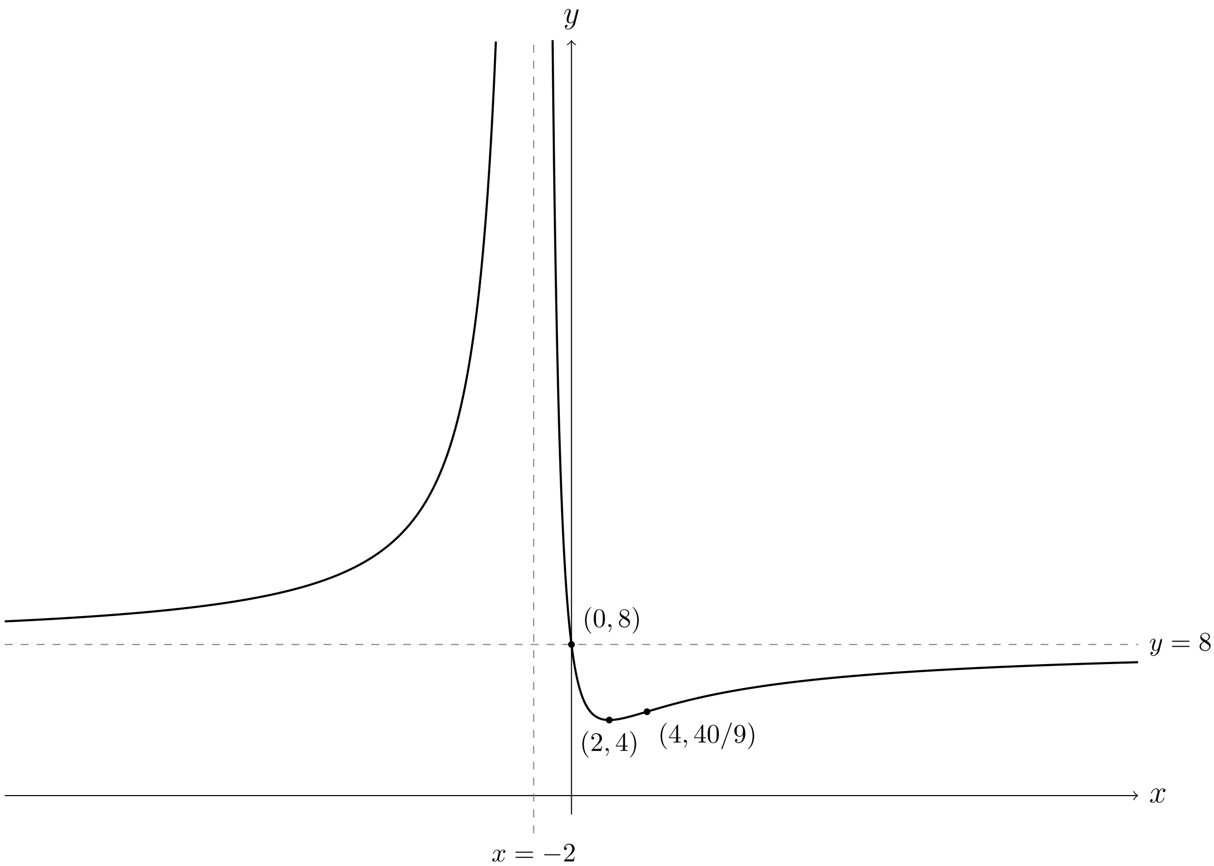
(d) Local minimum: $(2, 4)$. No local maximum.

(e) Concave down on $(4, \infty)$. Concave up on $(-\infty, -2)$, $(-2, 4)$.

(f) Inflection point: $(4, 40/9)$

(g) $(0, 8)$

(h)



16. (a) $e^2 x - 4\ln x + \frac{3}{8}x^{8/3} + C$

(b) $\frac{x^7}{7} + x^2 - \frac{1}{3x^3} + C$

(c) $\tan x + \sec x + C$

(d) $\sqrt{3} - 1 - \frac{\pi}{12}$

17. $\frac{2x^3}{1+x^4}$.

18. $\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{2i}{n}\right)^2 + 1\right]\frac{2}{n} = \frac{22}{3}$

19. -96