1. Evaluate the following integrals.

(a) $\int x^2 \cos^2(x^3)\,dx$

(b) $\int e^{3x} \sin(2x)\,dx$

(5) (c) $\int \dfrac{2x^2 - 3x - 3}{(x-1)(x^2 - 2x + 5)}\,dx$

(d) $\int \dfrac{dx}{x^4\sqrt{x^2 - 9}}$

(e) $\int_0^{\pi/4} 4\sec^4\theta \tan\theta\,d\theta$

(f) $\int_1^{16} \dfrac{dx}{\sqrt{x}(1 + \sqrt[4]{x})}$

(g) $\int \dfrac{\ln(2x)}{x \ln x}\,dx$

2. Evaluate the following improper integrals.

(a) $\int_0^1 \dfrac{\ln x}{\sqrt{x}}\,dx$

(b) $\int_{-\infty}^{\infty} \dfrac{dx}{x^2 + 9}$

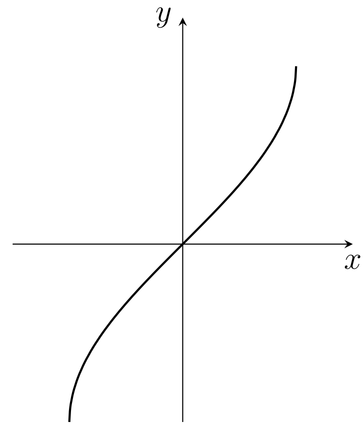
3. Evaluate the following limits.

(a) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1 - \sin x}{(2x - \pi)^2}$

(b) $\lim\limits_{x \to 2} \big(\sin(\pi/x)\big)^{\tan(\pi/x)}$

4. Let R be the region bounded by $y = \arcsin x$, $y = 0$ and $x = 1$.

(a) Evaluate the area of R



(b) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating R about the line $y = \pi/2$.

(c) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating R about the line $x = 2$.

5. Solve the differential equation

$$x\sqrt{1 - y^2} - \sqrt{1 - x^2}\,\frac{dy}{dx} = 0,$$

given that $y(1) = \sqrt{3}/2$. Express y as a function of x.

6. Find an equation of the curve passing through the point $(-2, e)$ that has the property that the slope of the tangent line at any of its points is equal to the product of the x- and y-coordinates of that point.

7. Determine whether the sequence $\{a_n\}$ converges or diverges. If the sequence converges find its limit; otherwise, explain why it diverges.

(a) $a_n = (-1)^n \dfrac{\sqrt{n} + 3}{5 - 3\sqrt{n}}$

(b) $a_n = n^2 \cos\left(\dfrac{1}{n}\right) - n^2$

8. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{\pi + (-2)^n}{3^n}$.

9. Determine whether the series converges or diverges. Justify your answer.

(a) $\frac{1}{3} - \cos(\frac{1}{3}) + \frac{1}{9} - \cos(\frac{1}{9}) + \frac{1}{27} - \cos(\frac{1}{27}) + \frac{1}{81} -$

$- \cos(\frac{1}{81}) + \cdots + 3^{-n} - \cos(3^{-n}) + \cdots$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{\sqrt[4]{n^9+8}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \ln\Big(1 + \frac{1}{n}\Big)$

10. Determine whether each of the following series is absolutely convergent, conditionally convergent or divergent. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{3^n n^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+2}+n}$

11. Find the interval of convergence of the power series $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{(x-2)^n}{n \ln n}$.

12. Find the Taylor series for $f(x) = \cos(x)$ centered at $a = \dfrac{\pi}{2}$.

13. Suppose that the power series $\displaystyle\sum_{n=1}^{\infty} c_n (x-2)^n$ converges if $x = -2$ and diverges if $x = -3$.

(a) Does the series converge when $x = 6$, or does it diverge, or could it either converge or diverge? Explain.

(b) Does the series $\displaystyle\sum_{n=1}^{\infty} c_n$ converge, or does it diverge, or could it either converge or diverge? Explain.

(c) Show that the series $\displaystyle\sum_{n=1}^{\infty} n c_n$ converges.

Answers:

1) a) $\frac{1}{6}[x^3 + \sin(x^3)\cos(x^3)] + C$;

b) $\frac{3}{13} e^{3x} \sin 2x - \frac{2}{13} e^{3x} \cos 2x + C$;

c) $-\ln|x-1| + \frac{3}{2}\ln|x^2 - 2x + 5| + \frac{1}{2}\tan^{-1}(\frac{x-1}{2}) + C$;

d) $\frac{1}{81}\left[\frac{\sqrt{x^2-9}}{x} - \frac{(\sqrt{x^2-9})^3}{3x^3}\right] + C$; e) 3;

f) $4 - 4\ln(\frac{3}{2})$; g) $\ln 2 \cdot \ln\big|\ln|x|\big| + \ln|x| + C$;

2) a) -4; b) $\pi/3$;

3) a) $1/8$; b) 1;

4) a) $\pi/2 - 1$;

b) $V = \pi \displaystyle\int_0^1 \left(\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{2} - \arcsin x\right)^2\right) dx$;

c) $V = 2\pi \displaystyle\int_0^1 (2-x)\arcsin x \, dx$;

5) $y = \sin(\frac{\pi}{3} - \sqrt{1-x^2})$; **6)** $y = e^{x^2/2-1}$;

7) a) D (the limits of a_n when n is even and when n is odd are different); b) $\lim\limits_{n\to\infty} a_n = -1/2$;

8) $\pi/6 + 4/15$;

9) a) D (TD); b) C (CT); c) D (LCT or treat it as a telescoping series);

10) a) AC; b) CC;

11) $x \in (1,3]$; **12)** $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$;

13) a) could either converge or diverge; (6 is the endpoint of the Interval of Convergence)

b) C ($x = 3 \in$ IoC); c) LCT with $\displaystyle\sum_{n=1}^{\infty} c_n 2^n$.

 This is the end of the examination.