1. (28 points) Evaluate the following integrals.

(a) $\displaystyle\int \frac{\sqrt{x} - 6x^5 + x^4 \cot(x^2)}{x^3}\, dx$

(b) $\displaystyle\int \frac{4x^2}{\sqrt{x^3 - 8}}\, dx$

(c) $\displaystyle\int_4^8 |7 - x|\, dx$

(d) $\displaystyle\int x^2 \ln(18x)\, dx$

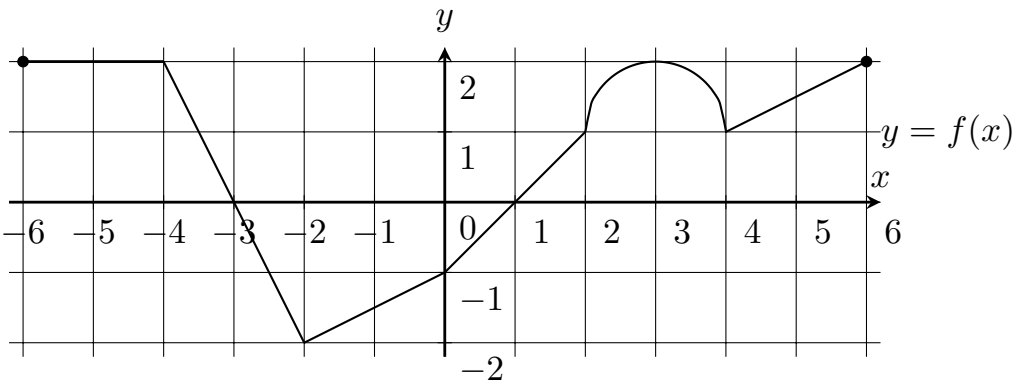
(e) $\displaystyle\int \frac{x^2 + 9x + 8}{x(x+2)^2}\, dx$

(f) $\displaystyle\int \cos(\cos x)\sin(x)\, dx$

(g) $\displaystyle\int \frac{\csc(x)\cot(x) - \sin(x)}{\sin(x)\cot(x)}\, dx$

2. (2 points) Give the value(s) of a and b that satisfy the equation $\displaystyle\int_5^a f(x)\, dx - \int_b^2 f(x)\, dx = \int_2^7 f(x)\, dx$

3. (6 points) Given the graph of function f below, find the following



(a) $\displaystyle\int_{-6}^0 f(x)\, dx$

(b) $\displaystyle\int_6^0 f(x)\, dx$

(c) $\displaystyle\int_{-6}^6 2f(x)\, dx$

4. (8 points) Given $\displaystyle\int_3^5 \frac{1}{x \ln x}\, dx$

(a) Use the trapezoidal rule with $n = 2$ to approximate the definite integral. Your answer should be accurate to 4 decimal places.

(b) Use substitution to evaluate the definite integral. Your answer should be accurate to 4 decimal places.

5. (4 points) Find the area of the region enclosed by $f(x) = x^2 - 1$ and $g(x) = -x^2 - 2x + 3$.

6. (4 points) The weekly demand function for the SoGreen coffee mug, a plant-based reusable mug, is $D(x) = \dfrac{144}{\sqrt{x+10}}$, and the supply function is $S(x) = \sqrt{x+10}$.

(a) Find the equilibrium point.

(b) Evaluate the consumers' surplus.

7. (4 points) Evaluate the following limits if possible.

(a) $\lim_{x\to 0} \dfrac{x-\sin(x)}{x^3}$ (b) $\lim_{x\to 1^+} \dfrac{\ln(x)-x+1}{x\ln(x)-1}$

8. (4 points) Solve the differential equation for y given $\dfrac{dy}{dx} = \dfrac{x^2y-4y}{x+2}$, with initial condition $y(0)=2$.

9. (5 points) The rate of decay at time t in hours of a radioactive substance N is proportional to the amount of substance present. If 75% of the initial amount of radioactive substance has decomposed after 10 hours, find the remaining amount of radioactive substance after 15 hours if initially the amount of radioactive substance is 300 g.

10. (10 points) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

(a) $\displaystyle\int_0^{\infty} \frac{e^{5x}}{e^{5x}+1}\,dx$ (b) $\displaystyle\int_{-\frac{1}{3}}^{0} \frac{3x+2}{\sqrt{3x^2+4x+1}}\,dx$

11. (2 points) Give the n^{th} term of the sequence $\left\{-\dfrac{1}{1}, \dfrac{3}{3}, -\dfrac{9}{5}, \dfrac{27}{7}, -\dfrac{81}{9}, \ldots\right\}$

12. (3 points) Consider the following sequence $a_n = \dfrac{3n(n-1)!}{n!}$

(a) Does the sequence converge? If so, find its limit.

(b) Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge of diverge? State which test you are using.

13. (2 points) Explain whether the following statement is true of false:

If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n \neq 0$.

14. (14 points) Determine the convergence or divergence of the following series. Mention the test you used. In the case of a convergent geometric or telescoping series, find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}+1}{n^4}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{2^{n+1}}{3^{n-1}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3n+1}$

15. (4 points) A deposit of \$150 is made at the beginning of each month for 5 years in an account that pays an annual rate of 2% interest compounded monthly. Find the total balance in this account at the end of 5 years.

ANSWERS

1. (a) $-\dfrac{2}{3\sqrt{x^3}} - 2x^3 + \dfrac{\ln|\sin(x^2)|}{2} + C$

(b) $\dfrac{8}{3}\sqrt{x^3-8} + C$

(c) 5

(d) $\dfrac{x^3}{3}\ln(18x) - \dfrac{x^3}{9} + C$

(e) $-\dfrac{3}{x+2} + 2\ln|x| - \ln|x+2| + C$

(f) $-\sin(\cos x) + C$

(g) $\ln|\cos x| - \cot x + C$

2. $a = 7$ and $b = 5$

3. (a) $= 1$

(b) $= -5 - \pi/2$

(c) $= 12 + \pi$

4. (a) ≈ 0.3941

(b) ≈ 0.3818

5. 9

6. (a) $(134, 12)$

(b) $\approx \$937.26$

7. (a) 1/6

(b) 0

8. $y = 2e^{\frac{x^2}{2}-2x}$

9. 37.5 g

10. (a) diverges

(b) converges, the value is 1

11. $(-1)^n \dfrac{3^{n-1}}{2n-1}$

12. (a) converges to 3

(b) div. by div. test

13. false, if the series converges, the limit of the general term must be 0.

14. (a) telescoping, conv. to 1/2

(b) conv. p series

(c) geometric series, conv. to 8

(d) div. by Ratio test

15. \$9472.87