1. Evaluate the following integrals.

(5) (a) $\displaystyle\int \frac{5x^2 - 3x + 10}{(x-2)(x^2+4)}\, dx$

(5) (b) $\displaystyle\int_0^{\pi/4} \frac{\sin^4 x}{\cos^6 x}\, dx$

(5) (c) $\displaystyle\int \frac{\sqrt{x^2-25}}{x^4}\, dx$

(5) (d) $\displaystyle\int e^{-x}\cos(3x)\, dx$

(5) (e) $\displaystyle\int_0^{9} \frac{1}{\sqrt{\sqrt{x}+1}}\, dx$

(5) (f) $\displaystyle\int_0^{\ln 3} \frac{e^x}{\sqrt{15+2e^x-e^{2x}}}\, dx$

(5) (g) $\displaystyle\int x \arcsin x\, dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to\pi} \frac{\sin^2(2x)}{1+\cos x}$

(3) (b) $\displaystyle\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan x\right)$

(3) (c) $\displaystyle\lim_{x\to 0^+} \left(1+\frac{x}{2}\right)^{4/x}$

(7) **3.** Determine whether the following improper integrals converge or diverge. If an integral converges, give its exact value.

(a) $\displaystyle\int_{-1}^{11} \frac{1}{\sqrt[3]{(x-3)^4}}\, dx$

(b) $\displaystyle\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$

(4) **4.** Solve the differential equation: $\sqrt{1-x^2}\,\dfrac{dy}{dx} = x + xy^2$, with initial condition $y(0) = 1$.

(3) **5.** A rumour starts in a town with a population of 1000. The rumour spreads at a rate proportional to the number of people who at time t (in weeks) have **not** heard the rumour. Initially, 25 people heard the rumour and at the end of 3 weeks, 675 people had heard it. How many people will have heard the rumour at the end of 6 weeks?

(6) **6.** Sketch $\mathcal{R}$, the region bounded by $y = \sin(x)$ and the x-axis with $x \in [0, \pi]$.
Set up, **but do not evaluate**, the integrals needed to find the volume of the solids of revolution obtained by revolving $\mathcal{R}$ about:

(a) the x-axis

(b) the line $x = -2$.

(c) the line $y = 2$.

(4) **7.** Find the length of the curve $y = \arcsin(x) + \sqrt{1-x^2}$ on its domain.

(4) **8.** (a) Does the series $\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$ converge? Justify your answer.

(b) Does the corresponding sequence $\left\{ \frac{3^n}{(2n)!} \right\}$ converge? Justify your answer.

9. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

(3) (a) $\sum_{n=1}^{\infty} \frac{\arctan n}{\sqrt[3]{n^2+1}}$

(3) (b) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1} - \frac{3}{2^{n-1}} \right)$

(3) (c) $\sum_{n=1}^{\infty} \frac{1}{3n + n\cos^2 n}$

(3) **10.** Find the sum of the following series:
$\sum_{n=1}^{\infty} \left(\cos^{-1}\left(\frac{1}{n}\right) - \cos^{-1}\left(\frac{1}{n+1}\right) \right)$

11. Determine whether the following series are absolutely convergent, conditionally convergent or or divergent. Justify your answer.

(4) (a) $\sum_{n=1}^{\infty} (-1)^n \frac{e^{1/n^2}}{n}$

(4) (b) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{3n+5} \right)^{2n}$

(4) **12.** Find the radius and interval of convergence of the power series
$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}}{4^n} (x-3)^n$

(4) **13.** Find the Maclaurin series expansion of $f(x) = \ln(x+2)$, and express the series using the appropriate sigma notation.

Answers

1. (a) $3\ln|x-2| + \ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

(b) $\frac{1}{5}$

(c) $\frac{1}{75}\left(\frac{\sqrt{x^2-25}}{x}\right)^3 + C$

(d) $\frac{1}{10}(3e^{-x}\sin(3x) - e^{-x}\cos(3x)) + C$

(e) $\frac{16}{3}$

(f) $\frac{\pi}{6}$

(g) $\frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C$

2. (a) 8

(b) 1

(c) e^2

3. (a) Divergent

(b) 2

4. $y = \tan\left(-\sqrt{1-x^2} + \frac{\pi+4}{4}\right)$

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6. (a) Disk Method; $v = \int_0^{\pi} \pi \sin^2 x \, dx$

(b) Shell Method; $v = 2\pi \int_0^{\pi} (x+2)\sin x \, dx$

(c) Washer Method; $v = \pi \int_0^{\pi} (2^2 - (2-\sin x)^2) \, dx$

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8. (a) Convergent by the Ratio Test

(b) Yes

9. (a) Divergent by L.C.T.

(b) Divergent by Test for Divergence

(c) Divergent by C.T.

10. $-\frac{\pi}{2}$

11. (a) Conditionally Convergent by A.S.T. and L.C.T.

(b) Absolutely Convergent by Root Test

12. Radius = 4 and Interval of Convergence $x \in (-1, 7)$

13. $\ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{2^n n}$