

1. (12 points) Evaluate the following limits.

(a) $\lim_{x \rightarrow 2^-} \frac{2x^3 - 4x^2}{3x^2 - 8x + 4}$

(b) $\lim_{x \rightarrow 0} \frac{\sin^2(3x)}{5x \sin(2x)}$

(c) $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 + 3x^5}}{2x^3 + \sqrt{9x^6 + 7x^5}}$

(d) $\lim_{x \rightarrow 2^+} \frac{\sqrt{x-2} - (x-2)}{6-3x}$

2. (4 points) Let

$$f(x) = \begin{cases} x^2 - k - 3, & x < -1 \\ k + 4, & x = -1 \\ k^2 + 4x - 4, & x > -1 \end{cases}$$

- (a) Find all values for k such that $\lim_{x \rightarrow -1} f(x)$ exists.

- (b) Find all values for k that make f continuous at all points.

3. (4 points) Let $f(x) = \frac{1}{3-2x}$. Use the limit definition of derivative to find $f'(x)$.

4. (16 points) Find $\frac{dy}{dx}$ for each of the following. Do not simplify your answer.

(a) $y = 16\sqrt[4]{x} + e^x - x^e + \frac{\pi}{x}$

(b) $y = \frac{(8-5x^2)^4}{\tan(7x) - 9}$

(c) $y = e^{\sqrt{2x^3}}$

(d) $y = (\sin x)^{4 \ln x}$

5. (4 points) Write an equation of the tangent line to the curve

$$x^2y + \sin y + \frac{4}{\pi}y = 3e^x$$

at the point $(0, \pi/2)$.

6. (6 points) Let θ (in radians) be an acute angle in a **right** triangle and let x and y be, respectively, the lengths of the sides adjacent and opposite to θ . Suppose also that x and y vary with time. At a certain instant, $x = 4$ cm and increasing at 8 cm/s, while $y = 3$ cm and is decreasing at 2 cm/s. How fast is θ changing at that instant?

7. (6 points) A box with a square base and open top needs to be made. The material for the base of the box costs \$10 per square meter, while the material for the sides cost \$5 per square meter. Using only \$120 what are the dimensions of such a box with largest volume?

8. (6 points) Find the absolute extrema of $f(x) = \frac{x}{2} + \frac{2}{x^2}$ on the interval $[1, 4]$.

9. (6 points) The function $s(t) = t^3 - 3t^2$ describes the position of a particle moving along a coordinate line, where s is in meters and $t \geq 0$ is in seconds.

- (a) Find the velocity function.
- (b) At what times is the particle at rest?
- (c) When is the particle moving in the positive direction?

10. (10 points) Consider the following function, along with its two first derivatives.

$$f(x) = \frac{x+2}{\sqrt{x^2+2}}, \quad f'(x) = \frac{2(1-x)}{(x^2+2)^{3/2}}, \quad f''(x) = \frac{2(x-2)(2x+1)}{(x^2+2)^{5/2}}.$$

(It might help to know that $f(-1/2) = 1$, $f(0) \approx 1.41$, $f(1) \approx 1.73$, and $f(2) \approx 1.63$.)

- (a) Find the domain and intercepts of f .
 - (b) Find the vertical and horizontal asymptotes of f (if any).
 - (c) Find the intervals of increase/decrease of f .
 - (d) Find the local (relative) extrema of f .
 - (e) Find the intervals of concavity of f .
 - (f) Find all points of inflection of f .
 - (g) On the next page, sketch a graph of f .
11. (16 points) Evaluate each of the following integrals.

(a) $\int \left(\frac{2}{x} - \sqrt[3]{x^5} + 7e^x \right) dx$

(b) $\int \frac{(5x-3)^2}{x} dx$

(c) $\int \frac{1 - \sin \theta}{\cos^2 \theta} d\theta$

(d) $\int_2^3 \frac{x^2 + 8x + 15}{x+3} dx$

12. (4 points) Given $f(x) = \int_6^{1/x} \frac{t}{\sqrt{1+t}} dt$, find:

- (a) $f(1/6)$
- (b) $f'(x)$

13. (4 points) Express $\int_0^5 \sin(x^2) dx$ as the limit of a Riemann sum. Do not evaluate the limit.

14. (2 points) Decide whether the equality below is correct or not. Justify.

$$\int \ln x \, dx = x \ln x - x + C$$

Answers:

1. (a) 2
 (b) $9/10$
 (c) 2
 (d) $-\infty$
2. (a) 2, -3
 (b) -3
3. $2/(3 - 2x)^2$
4. (a) $4x^{-3/4} + e^x - ex^{e-1} - \pi/x^2$
 (b) $\frac{4(8-5x^2)^3(-10x)(\tan(7x)-9)-(8-5x^2)^4(7\sec^2(7x))}{(\tan(7x)-9)^2}$
 (c) $e^{\sqrt{2x^3}} 3x^2 / \sqrt{2x^3}$
 (d) $(\sin x)^{4 \ln x} \left[\frac{4}{x} \ln(\sin x) + \frac{4 \ln x \cos x}{\sin x} \right]$
5. $y = \frac{3\pi}{4}x + \frac{\pi}{2}$
6. $-32/25 \text{ rad/s}$
7. $2 \times 2 \times 2$
8. abs. max. = 2.5 / abs. min. = 1.5
9. (a) $v(t) = 3t^2 - 6t$
 (b) $t = 0, t = 2$
 (c) when $t > 2$
10. (a) domain: \mathbb{R} ; x -int.: $(-2, 0)$; y -int.: $(0, \sqrt{2})$
 (b) h.a.: $y = -1, y = 1$; v.a.: none
 (c) inc.: $(-\infty, 1)$; dec.: $(1, \infty)$
- (d) local max. at $x = 1$; no local min.
 (e) conc. up: $(-\infty, -1/2), (2, \infty)$; conc. down: $(-1/2, 2)$
 (f) inflection pts at $x = -1/2, x = 2$
- (g)
11. (a) $2 \ln |x| - \frac{3}{8}x^{8/3} + 7e^x + C$
 (b) $\frac{25}{2}x^2 - 30x + 9 \ln |x| + C$
 (c) $\tan \theta - \sec \theta + C$
 (d) $15/2$
12. (a) 0
 (b) $\frac{1/x}{\sqrt{1+1/x}} \frac{-1}{x^2}$
13. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin(25i^2/n^2) 5/n$
14. Correct. (Because $[x \ln x - x + C]' = \dots = \ln x$.)