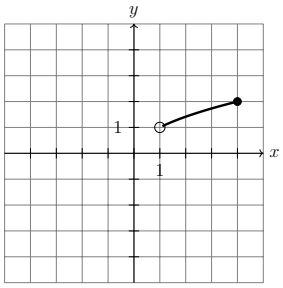
1 Multiple choice

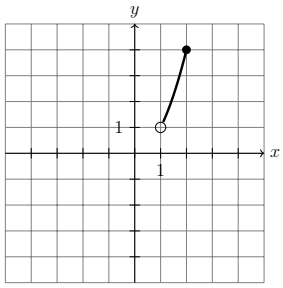
For each of the following questions, there is only one correct answer. Circle your choice. If two choices are selected for the same question, no marks will be awarded.

1. Consider the four graphs shown below.

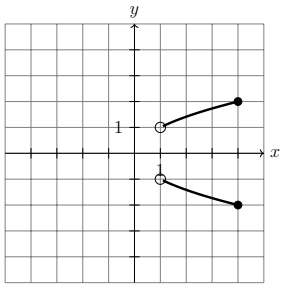
I



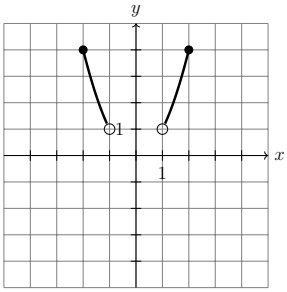
II



III



IV



(a) [1 point] How many of these graphs are functions?

A. 0 B. 1 C. 2 D. 3 E. 4

(b) [1 point] How many of these graphs are invertible functions?

A. 0 B. 1 C. 2 D. 3 E. 4

(c) [1 point] Which of these graphs is a function with range $(1, 4]$?

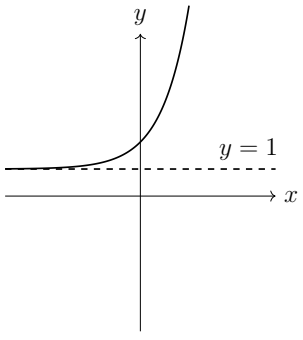
A. Only I B. Only II C. I and III D. II and IV E. None of the above

2. [1 point] Suppose two similar objects have volumes 16 cm^3 and 2 cm^3. If the larger object has surface area 36 cm^2, what is the surface area of the smaller object?

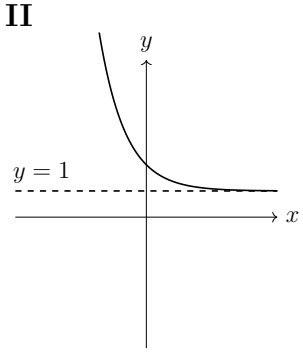
A. 4.5 cm^2 B. 9 cm^2 C. 18 cm^2 D. 72 cm^2 E. 144 cm^2

3. Consider the four graphs shown below.

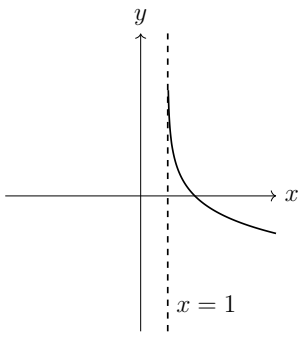
I



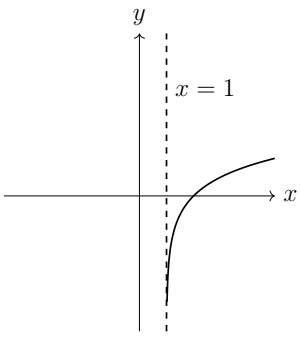
II



III



IV



(a) [1 point] Which of these could be the graph of $y = \ln(x - 1)$?

A. I B. II C. III D. IV E. None of the above

(b) [1 point] Which of these could be the graph of $y = \sqrt{x - 1}$?

A. I B. II C. III D. IV E. None of the above

(c) [1 point] Which of these could be the graph of $y = 1 + e^{-x}$?

A. I B. II C. III D. IV E. None of the above

4. [1 point] Suppose θ is an angle in standard position such that $\tan\theta < 0$ and $\sin\theta < 0$. In which quadrant will we find the terminal side of θ?

A. I B. II C. III D. IV E. None of the above.

5. [1 point] Which of the following vectors is perpendicular to $\langle 9, 6\rangle$?

A. $\langle 6, 4\rangle$ B. $\langle 4, 6\rangle$ C. $\langle -6, 4\rangle$ D. $\langle -4, 6\rangle$

6. [1 point] When solving triangles, in which configuration is it possible to have two solutions?

A. Side-Side-Side

B. Side-Angle-Angle

C. Side-Angle-Side

D. Angle-Side-Side

E. Angle-Angle-Angle

7. [1 point] For which value of c below does the equation $x^2 + 2x + c = 0$ have no solution?

A. $c = 0$ B. $c = 1$ C. $c = -1$ D. $c = 2$ E. $c = -2$

8. [1 point] If $f(x) = \dfrac{1}{x-2}$, then $f^{-1}(x)$ is which of the following?

A. $y = x - 2$ B. $y = \frac{1}{x} + 2$ C. $y = \frac{1}{x} - 2$ D. $y = \frac{1}{x+2}$ E. $y = \frac{1}{x-2}$

9. [1 point] The domain of $y = \dfrac{x^2 - 4x}{x^2 - 16}$ is which of the following?

A. $\mathbb{R}\backslash\{4\}$ B. $\mathbb{R}\backslash\{-4\}$ C. $\mathbb{R}\backslash\{-4, 4\}$ D. $\mathbb{R}\backslash\{-4, 0, 4\}$ E. $(-\infty, -4) \cup (4, \infty)$

2 Short Answer

10. Determine if the following statements are true or false for all values of x in the domain of both sides. Answer "**True**" or "**False**".

(a) [1 point] $(x + 2)^3 = x^3 + 8$ _____

(b) [1 point] $\left(\sqrt{x^5 y}\right)^6 = x^{15} y^3$. _____

(c) [1 point] $\ln\left(\dfrac{1}{\sqrt[5]{x}}\right) = -\frac{1}{5}\ln x$. _____

(d) [1 point] $\log(Ax^3) = 3\log(Ax)$. _____

11. Factor completely:

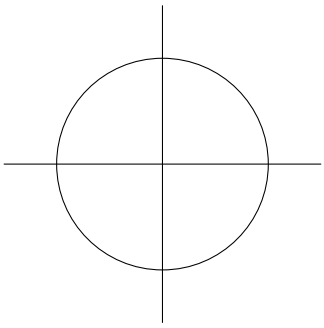
(a) [2 points] $3x^2 - 11x - 20$

(b) [3 points] $x^4 - 10000$

12. [2 points] If $\sin\theta = \frac{3}{7}$ and θ is in the first quadrant, compute $\sec\theta$.

13. [2 points] Convert $140°$ to radians.

14. (a) [2 points] Sketch the angle $\theta = \frac{11\pi}{6}$ in standard position on the circle below.



(b) [2 points] Find the exact value of $\tan\left(\frac{11\pi}{6}\right)$.

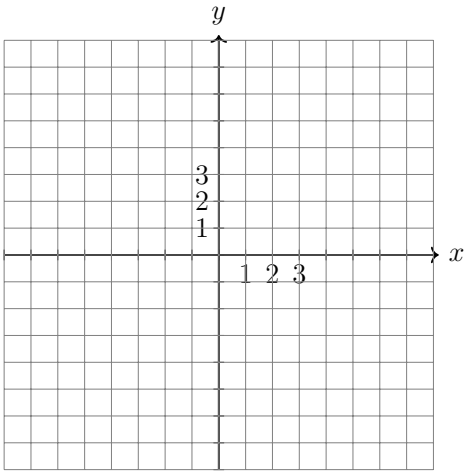
15. [2 points] Find the future value after 8 years of \$3000 invested at 6% interest compounded monthly.

16. [2 points] Evaluate $\log_3(150)$ to four decimal places.

17. [2 points] Given the vectors $\vec{u} = \langle 7, -2\rangle$ and $\vec{w} = \langle 4, -5\rangle$, compute $2\vec{u} - 3\vec{w}$.

18. Given the vector $\vec{v} = \langle 2, -5\rangle$,

(a) [2 points] Sketch $\vec{v}$ in the grid below:



(b) [2 points] Find a unit vector in the same direction as $\vec{v}$.

3 Long Answer

19. [5 points] Consider the function $f(x) = \begin{cases} 6 - 2x \text{ if } -1 < x \leqslant 3 \\ \frac{1}{3}x - 2 \text{ if } 3 < x < 9 \end{cases}$

(a) Evaluate $f(3)$.

(b) Sketch a graph of $f(x)$.

(c) Use your graph to state the range of f in interval notation.

20. [4 points] Solve for x: $30x^4 + 10x^3 = 270x^2 + 90x$

21. [3 points] Use long division to divide: $\dfrac{6x^3 + 7x^2 + 7x + 13}{3x + 5}$

(Express your answer in the form $Q(x) + \dfrac{R(x)}{D(x)}$.)

22. [4 points] Subtract and simplify: $\dfrac{3}{(x+4)(x-5)} - \dfrac{1}{(x-5)(x-2)}$

23. [3 points] Given $f(x) = \frac{1}{x}$ and $g(x) = \dfrac{5x+3}{x^2+2}$, find and simplify $g(f(x))$.

24. [3 points] Solve the following inequality: $\dfrac{x+2}{(x-4)(x-1)} \leqslant 0$

25. [3 points] State the domain of the function: $f(x) = \sqrt{x^2 - 9}$

26. [2 points] Find the inverse of the function: $f(x) = 5e^{(x+7)}$

27. [4 points] Solve for x: $\log_2(x - 14) = 5 - \log_2(x)$

28. [3 points] Find all θ in $[0°, 360°)$ that satisfy the equation: $\cos\theta = -0.3$ (Two decimal places.)

29. [3 points] Find all θ in $[0, 2\pi)$ that satisfy $\csc\theta = -\sqrt{2}$. (Exact Value.)

30. [4 points] Prove the identity: $\dfrac{\tan x}{1 - \sec x} = \dfrac{1}{\cot x - \csc x}$

31. [4 points] A triangle has sides of length $a = 9, b = 6$, and $c = 4$ across from angles of measure A, B, C respectively. Find angles A, B, and C to three decimal places.

32. [3 points] Sketch a graph of $y = -3\cos(x/4)$ on the given axes.

4 Applications

33. [5 points] Suppose that a ball shot up into the air from the surface of Planet Z has height $y = -5t^2 + 30t$ metres after t seconds.

(a) Sketch a graph of the height of the ball below.

(b) At what time does the ball return to ground level on Planet Z?

(c) At what time does the ball reach its maximum height?

(d) What is the maximum height reached by the ball?

34. [4 points] A flagpole has two flags on it: a red flag at the top and a blue flag lower down. Measured from a point on the ground the distance to the red flag *at the top* is 8m. From the same point on the ground the angles of elevation to the flags are 58° and 46°. How far apart are the two flags?

35. The number of cells in a colony of bacteria is given by $P(t) = 10(\sqrt[3]{2})^t$, where t is measured in weeks.

(a) [1 point] How many cells are in the colony at time $t = 0$?

(b) [2 points] At what time t will the population reach 1000 cells? (Round to two decimal places.)

ANSWERS: 1(a) D 1(b) C 1(c) D 2. B 3(a) D 3(b) E 3(c) B 4. D 5. D. 6. D. 7. D. 8. B. 9. C. 10 (a) False 10(b) True 10(c) True 10(d) False

11(a) $(3x+4)(x-5)$ 11(b) $(x-10)(x+10)(x^2+100)$ 12. $\dfrac{7}{2\sqrt{10}}$ 13. $\dfrac{7\pi}{9}$ 14(a) Sketches at end.

14(b) $-\dfrac{1}{\sqrt{3}}$ 15. \$4842.43 16. 4.5609 17. $\langle 2, 11\rangle$ 18(a) Sketches at end. 18(b) $\left\langle \dfrac{2}{\sqrt{29}}, \dfrac{-5}{\sqrt{29}} \right\rangle$

19(a) 0 19(b) Sketches at end. 19(c) $(-1, 8)$ 20. $x = -3, -1/3, 0, 3$ 21. $2x^2 - x + 4 + \frac{-7}{3x+5}$

22. $\dfrac{2}{(x+4)(x-2)}$ 23. $\dfrac{5x+3x^2}{1+2x^2}$ 24. $(-\infty, -2] \cup (1, 4)$ 25. $(-\infty, -3] \cup [3\infty)$ 26. $\ln(x/5) - 7$

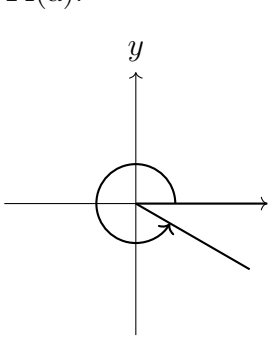
27. 16 28. $\theta = 107.498°,\ 252.54°$ 29. $\theta = 5\pi/4,\ 7\pi/4$ 30. Multiply top and bottom by $\cot x$.

31. $C = 20.742°, B = 32.089°, A = 127.169°$ 32. Sketches at end. 33(a) Sketches at end.

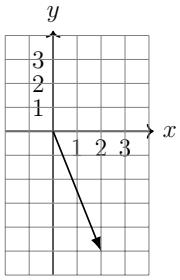
33(b) 6 seconds. 33(c) 3 seconds 33(d) 45m 34. 2.394m 35. 10 cells 19.93 weeks.

SKETCHES:

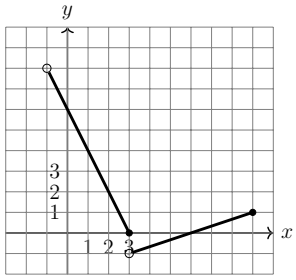
14(a).



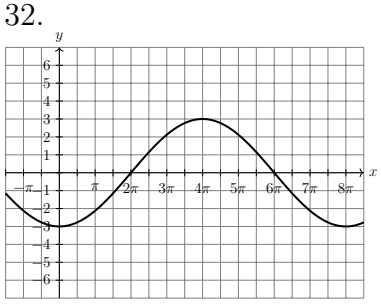
18(a).



19(b).



32.



33.