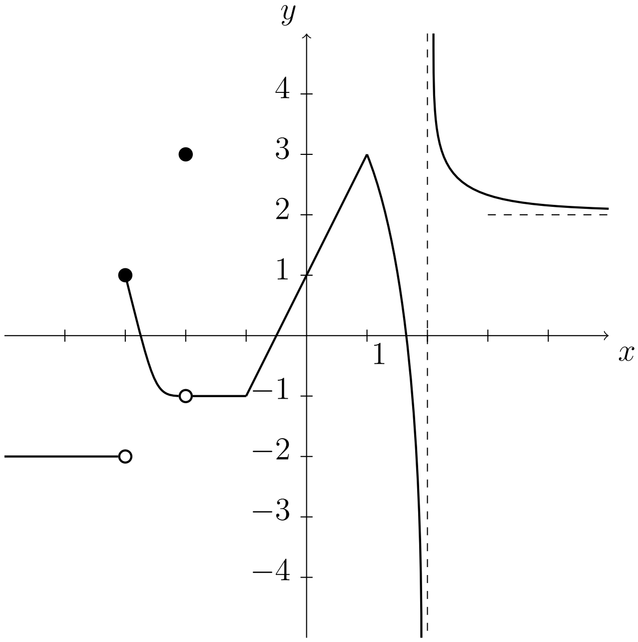
1. (3 points) Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.



(a) $f(-2) =$

(b) $\lim_{x\to 2^-} f(x) =$

(c) $\lim_{x\to -2} f(x) =$

(d) $\lim_{x\to -3^+} f(x) =$

(e) $\lim_{x\to 2} f(x) =$

(f) $\lim_{x\to -\infty} f(x) =$

2. (8 points) Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.

(a) $\lim_{x\to 3} \dfrac{\frac{1}{x+3} - \frac{1}{2x}}{x^2 - 9}$

(b) $\lim_{x\to 4^-} \dfrac{2x^2 - 7x - 4}{x^2 - 8x + 16}$

3. (5 points) Given

$$f(x) = \begin{cases} x^2 - x & x < 3 \\ \dfrac{x^2 + 5x}{x - 5} & x \geqslant 3, \end{cases}$$

find the value(s) of x where the function is not continuous and justify your answers.

4. (14 points) Find the derivative of each of the following functions. Do not simplify your answers.

(a) $f(x) = \sec(9x)$ (2 points)

(b) $f(x) = (4x+1)^3 \cos(7x^2 - 6x)$ (4 points)

(c) $f(x) = \dfrac{3^{x^2} - 8x}{5 + \tan^6(x)}$ (4 points)

(d) $f(x) = 8x^{\ln(x)}$ (4 points)

5. (4 points) Use logarithmic differentiation to find the derivative of $y = \dfrac{\sqrt{2x+5}}{3^x (x+4)^7}$.

6. (5 points) Given $e^{xy} = x + 2y$

(a) Find $y' = \dfrac{dy}{dx}$.

(b) Find an equation of the tangent line at the point $(x, y) = (1, 0)$.

7. (6 points) Find the absolute extrema of the function $g(t) = t - 9\sqrt[3]{t}$ on the interval $[-1, 5]$.

8. (10 points) Given

$$f(x) = \frac{(x+1)}{(x+2)^2} \qquad f'(x) = \frac{-x}{(x+2)^3} \qquad f''(x) = \frac{2(x-1)}{(x+2)^4}$$

(a) Find the domain of f,

(b) Find the x- and y-intercepts of f,

(c) Find any vertical and horizontal asymptotes of f,

(d) Find the intervals of increase and decrease of f,

(e) Find any local extrema of f,

(f) Find the intervals of concavity of f,

(g) Find any points of inflection of f,

(h) Use your answers from the previous parts to sketch a graph of f on the grid below. Choose the scale of your axes carefully. Show all relevant information on the graph.

9. (6 points) A cooking school charges \$300 per student for a series of courses if exactly 12 sign up. However, if more than 12 students sign up, then each tuition is reduced by \$6 for each additional student. Note that the maximum enrolment is 62 and if fewer than 12 students sign up, then the courses are cancelled.

(a) How many students should be enrolled in the cooking school to maximize the revenue?

(b) What would be the tuition per student in this case?

10. (4 points) The demand function of the new waterproof SoundDrop speaker is given by $x = 300 - p^2$ where x is the quantity demanded and p is the unit price.

(a) Find the price elasticity of demand function.

(b) Is the demand elastic or inelastic when $p = \$15$?

(c) Based on your answer in part (b), how, if at all, should the company modify its price to increase the revenue? Explain briefly.

Answers

1. (a) 3 (b) $-\infty$ (c) -1 (d) 1 (e) DNE (f) -2

2. (a) $\frac{1}{216}$ (b) $-\infty$

3. $f(x)$ is discontinuous at $x = 5$ and $x = 3$.

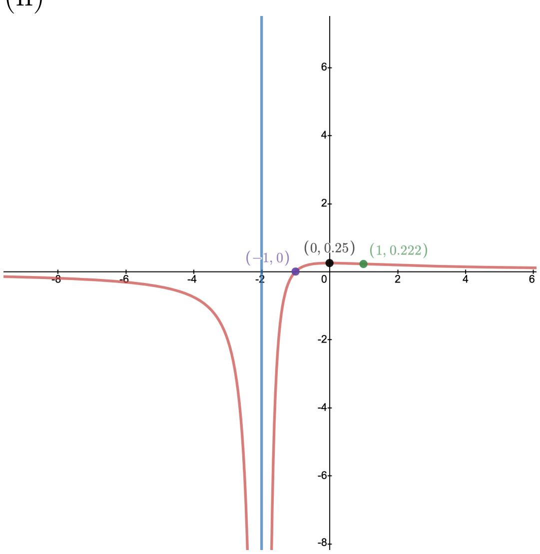
4. (a) $f'(x) = 9\sec(9x)\tan(9x)$
   (b) $f'(x) = 12(4x+1)^2\cos(7x^2-6x) - (4x+1)^3\sin(7x^2-6x)(14x-6)$
   (c) $f'(x) = \frac{(3^{x^2}2x-8)(5+\tan^6(x)) - 6(3^{x^2}-8x)\tan^5(x)\sec^2(x)}{\left(5+\tan^6(x)\right)^2}$
   $\cdot \ln 3$
   (d) $f'(x) = \frac{16}{x}x^{\ln(x)}\ln(x)$

5. $y' = \frac{\sqrt{2x+5}}{3^x(x+4)^7}\left[\frac{1}{2x+5} - \ln(3) - \frac{7}{x+4}\right]$

6. (a) $y' = \frac{1-ye^{xy}}{xe^{xy}-2}$ (b) $y = -x+1$

7. Critical numbers: $t = 0$, absolute max: $f(-1) = 8$ at $x = -1$, absolute min: $f(5) \approx -10.39$ at $x = 5$.

8. (a) $(-\infty, -2) \cup (-2, \infty)$
   (b) x-intercept $(-1, 0)$ y-intercept $(0, \frac{1}{4})$
   (c) Vertical asymptote: $x = -2$ Horizontal asymptote: $y = 0$
   (d) f is increasing on $(-2, 0)$ and decreasing on $(-\infty, -2) \cup (0, \infty)$
   (e) f has a local max at $(0, \frac{1}{4})$
   (f) f is concave up on $(1, \infty)$ and concave down on $(-\infty, -2) \cup (-2, 1)$.
   (g) f has a point of inflection $(1, \frac{2}{9})$
   (h)



9. (a) 31 students (b) \$ 186

10. (a) $E(p) = \frac{2p^2}{300-p^2}$
    (b) The demand is elastic ($E(15) = 6 > 1$)
    (c) Since the demand is elastic, the company should reduce the price to increase the revenue.