Final Examination
Calculus II
201-203-RE
16 December 2020

1. [**16** = **4+4+4+4**] Evaluate each integral.

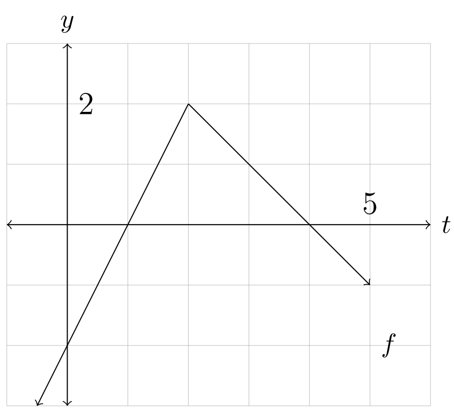
   (a) $\displaystyle\int_0^1 (12x-10)e^{3x^2-5x}\,dx$

   (b) $\displaystyle\int x\sec^2 x\,dx$

   (c) $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$

   (d) $\displaystyle\int \frac{2x^4-3x+12}{x^3-x^2}\,dx$

2. [**2**] The graph of f is given below. Find $\displaystyle\int_0^5 f(x)\,dx$.



3. [**4**] Solve the differential equation $e^{-x^2}y' = \dfrac{x}{y}$ with initial condition $y(0) = -3$.

4. [**5**] Centax is a special wax centaurs use on their manes as a sort of armor. The Centaur army has been called to war, and as such, the price of Centax is increasing at a rate proportional to the product of price and time. The price of Centax, $P(t)$, starts off at \$40 per pound of wax, but it is expected to double and reach a price of \$80 in just 2 days.

   (a) Clearly write the differential equation representing this problem as well as the initial conditions.

   (b) Find the function $P(t)$ for the price of Centax after t days (in its most simplified form).

   (c) How high will the price be after 4 days?

5. [**2**] Evaluate $\displaystyle\lim_{x\to 0}\frac{\ln|\cos(x)|}{\sin x}$ or explain why it does not exist.

6. [**2**] Find the general term of the sequence $\left\{\dfrac{4}{5}, \dfrac{9}{8}, \dfrac{16}{11}, \dfrac{25}{14}, \dfrac{36}{17}, \ldots\right\}$.

7. [**1**] If $a_n > 0$ and $\displaystyle\lim_{n\to\infty} a_n = 8$, does the sequence $b_n = (-1)^n a_n$ converge or diverge? Briefly explain your answer.

8. [**3=2+1**] For each sequence, find the limit or explain why it diverges.

   (a) $a_n = \dfrac{3n(n+1)!}{(n+2)!}$

   (b) $a_n = (-1)^n + 6$

9. [**9=5+4**] Evaluate each improper integral.

   (a) $\displaystyle\int_e^\infty \frac{\ln x}{x^2}\,dx$

   (b) $\displaystyle\int_2^4 \frac{6x}{x^2-4}\,dx$

10. [**13=4+3+3+3**] Determine the convergence or divergence of the following series. Mention any test you use. In the case of a convergent geometric or telescoping series, find the sum.

    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{n^2+8n+15}$

    (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5^n}{(2n)!}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sqrt{n}}{n^2}$

    (d) $\displaystyle\sum_{n=0}^{\infty} \frac{5^{n-1}}{2^{3n+1}}$

11. [**4**] Johnny Ontario wants to save money to buy a self-driving electric car. He predicts that 10 years from now, the car will cost \$20,000. To reach his goal, how much should he invest monthly if the annual interest rate is 3% compounded monthly?

12. [**1**] Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series and let S_n be its sequence of partial sums where $\displaystyle\lim_{n\to\infty} S_n = 7$. Find $\displaystyle\lim_{n\to\infty} a_n$.

(TOTAL = 62 MARKS)

ANSWERS

1. (a) $\frac{2}{e^2} - 2$
   (b) $x \tan x + \ln|\cos x| + C$
   (c) $2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$
   (d) $-9\ln|x| + x^2 + 2x + \frac{12}{x} + 11\ln|x-1| + C$
2. $\frac{3}{2}$
3. $y = -\sqrt{e^{(x^2)} + 8}$
4. (a) $\frac{dP}{dt} = kPt, \quad P(0) = 40,\ P(2) = 80.$
   (b) $P(t) = 40 \cdot 2^{t^2/4}$
   (c) \$640
5. 0
6. $a_n = \frac{(n+1)^2}{3n+2}$
7. $\lim a_n \neq 0$ so $\lim b_n$ does not exist (oscillating).
8. (a) 3
   (b) Does not exist (oscillating).
9. (a) $\frac{2}{e}$
   (b) Divergent ($+\infty$)
10. (a) Converges to $\frac{7}{12}$ (telescoping).
    (b) Convergent (ratio test).
    (c) Convergent (p-series).
    (d) Converges to $\frac{4}{15}$ (geometric).
11. \$142.76
12. 0