1. (16 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = \dfrac{1}{5\sqrt{x^3}} + \dfrac{e^2}{\sec(x)} - 7^x + \log_2(x)$

(b) $y = \tan^3\left(\dfrac{2x}{x^2+1}\right)$

(c) $y = (x - \sin(x))^{\ln(x)}$

(d) $\cot(y^2) - 6x^2y = 20$

2. (3 points) Suppose that $f(x) = e^x \cos(x)g(x), g(0) = 3$ and $g'(0) = -8$. Find $f'(0)$.

3. (3 points) Find the horizontal asymptotes of the graph of $f(x) = \dfrac{3 \cdot 2^x + 4}{2^x - 32}$.

4. (4 points) Find the critical numbers of $f(x) = 3(x+5)(x^2-5)^{1/3}$.

5. (5 points) The new John Abbott satellite is launched vertically into the air. Students are observing the launch on the ground 5 km from the point where the satellite is launched. At the moment when the satellite is 12 km high, it is travelling at 390 km/h. Determine the rate at which the distance between the satellite and the students is changing at that moment.

6. (11 points) Consider the following function, along with its first two derivatives.

$$f(x) = \frac{2(x-2)(2x-1)}{(x+1)^2} \quad , \quad f'(x) = \frac{18(x-1)}{(x+1)^3} \quad , \quad f''(x) = \frac{36(2-x)}{(x+1)^4}$$

(a) Find the domain and intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals of increase/decrease of f.

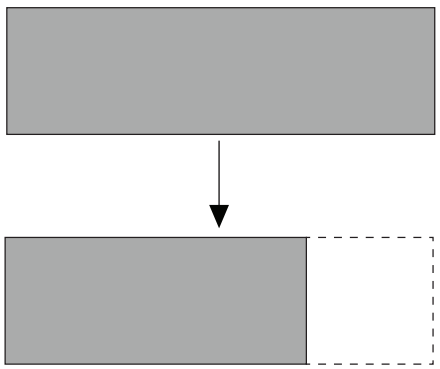
(d) Find the relative extrema of f.

(e) Find the intervals of concavity of f.

(f) Find all points of inflection of f.

(g) Sketch a graph of f, including all relevant points and asymptotes.

7. (5 points) A rectangle will have a square removed from the end (as shown). What is the maximum area of the rectangle that will remain, if the initial rectangle must have a perimeter of 24 cm?



8. (12 points) Evaluate the following integrals.

(a) $\displaystyle\int (x-3)\left(\sqrt{x} - \frac{2}{x}\right) dx$

(b) $\displaystyle\int \frac{\tan\theta - 2\sec\theta}{\cos\theta}\, d\theta$

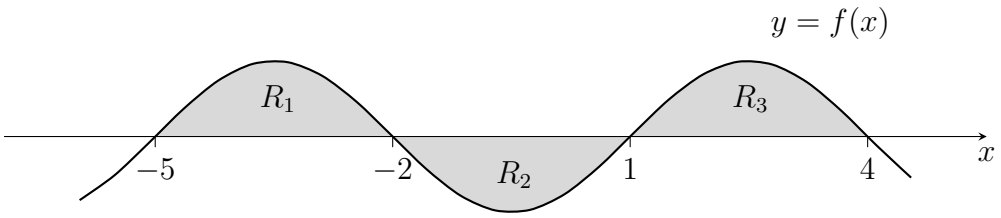
(c) $\displaystyle\int_1^4 \left(\frac{1}{4\sqrt{x}} + \frac{2}{x^2}\right) dx$

(d) $\displaystyle\int_0^3 |4 - 2x|\, dx$

9. (4 points) Compute $\int_0^3 (x^2+2)\,dx$ by first expressing it as a limit of Riemann sums. The following formulas are provided:

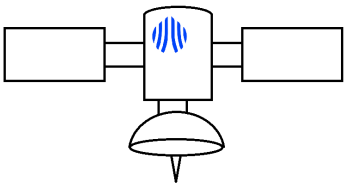
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 .$$

10. (3 points) The figure below shows the graph of a function f, in which the shaded regions R_1, R_2 and R_3 all have the same area.



Given that $\int_{-5}^{4} f(x)\,dx = 5$, find $\int_{-2}^{1} \big(2f(x)-1\big)\,dx$.

11. (2 points) Evaluate $g'(x)$ where $g(x) = \int_{\cos x}^{\pi} e^t \ln(t)\,dt$



Answers

1.

(a) $\dfrac{dy}{dx} = -\dfrac{3}{10x^{5/2}} - e^2 \sin x - 7^x \ln 7 + \dfrac{1}{x \ln 2}$

(b) $\dfrac{dy}{dx} = 3\left(\tan\left(\dfrac{2x}{x^2+1}\right)\right)^2 \sec^2\left(\dfrac{2x}{x^2+1}\right) \dfrac{2(x^2+1)-4x^2}{(x^2+1)^2}$

(c) $\dfrac{dy}{dx} = (x-\sin x)^{\ln x}\left(\dfrac{\ln(x-\sin x)}{x} + \dfrac{\ln x(1-\cos x)}{x-\sin x}\right)$

(d) $\dfrac{dy}{dx} = \dfrac{-12xy}{2y\csc^2(y^2)+6x^2}$

2. $f'(0) = -5$

3. $y = -\dfrac{1}{8}, y = 3$

4. $x = -\sqrt{5},\ x = -3,\ x = 1,\ x = \sqrt{5}$

5. The distance between the satellite and the students is increasing at a rate of 360 km/h at that moment.

6.

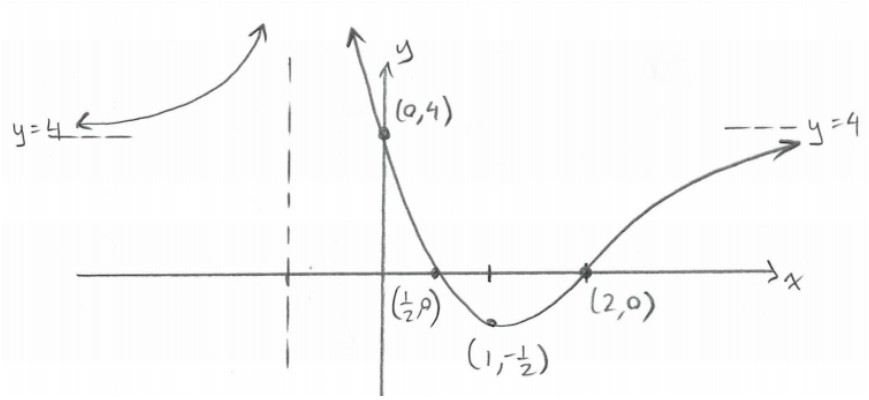
(a) $D : \mathbb{R} \setminus \{-1\}$, y-int : $(0, 4)$, x-int : $(1/2, 0)$ and $(2, 0)$.

(b) VA : $x = -1$, HA : $y = 4$.

(c) Increasing on $(-\infty, -1) \cup (1, \infty)$ and decreasing on $(-1, 1)$. There's a relative minimum at $(1, -1/2)$.

(d) Concave up on $(-\infty, -1) \cup (-1, 2)$ and concave down on $(2, \infty)$. There's a point of inflection at $(2, 0)$.

(e)



7. The maximum area of the rectangle that will remain is of 18 cm^2.

8.

(a) $\dfrac{2}{5}x^{5/2} - 2x - 2x^{3/2} + 6\ln|x| + C$

(b) $\sec\theta - 2\tan\theta + C$

(c) 2

(d) 5

9. 15

10. -13

11. $g'(x) = e^{\cos x}\ln(\cos x)\sin x$