JOHN ABBOTT CEGEP/COLLEGE

Department of Mathematics

Final Examination

December 14, 2020
09:00-11:00

CALCULUS II
201-NYB-05

Instructors:

Student name: ______________________

Student number: ______________________

Instructor: ______________________

Instructions

1. Write all of your solutions in this booklet and show all supporting work.
2. If the space provided is not sufficient, continue the solution on the opposite page.
3. Check that this booklet contains 2 pages, excluding this cover page.
4. Remember that the use of a calculator is not permitted.

1. Evaluate the following integrals.

(5) (a) $\displaystyle\int \frac{2x+3}{(x-1)(x^2+4)}\,dx$

(5) (b) $\displaystyle\int \frac{x}{\sqrt{6x-x^2}}\,dx$

(5) (c) $\displaystyle\int_0^{\pi/3} x\sec^2 x\,dx$

(5) (d) $\displaystyle\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

2. Evaluate the following limits.

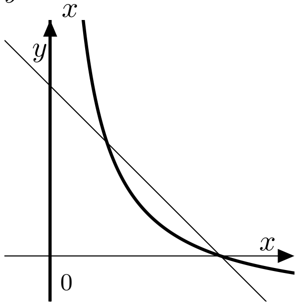
(3) (a) $\displaystyle\lim_{x\to 0} \frac{x^2 e^x}{\tan^2 x}$

(3) (b) $\displaystyle\lim_{x\to\infty} x^{1/\sqrt{x}}$

(4) **3.** Determine whether the following improper integral converges or diverges. If the integral converges, give its exact value.

$$\int_0^{\pi/4} \frac{\sec^2\theta}{\sqrt{1-\tan\theta}}\,d\theta$$

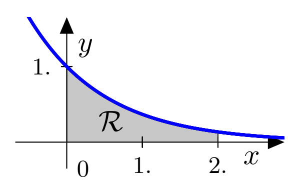
(3) **4.** Set up, but **do not evaluate** the integral required to find the area enclosed by the curves, $y = 3-x$ and $y = \dfrac{3}{x} - 1$.



5. Let $\mathcal{R}$ be the region bounded by $y = e^{-x}$, $x = 0$ and $x = 2$. Set up, but **do not evaluate** the integral for the volume obtained by rotating the region $\mathcal{R}$ about:

(2) (a) the $y-$axis.

(3) (b) the line $y = 3$.



(4) **6.** Solve the following differential equation, $\dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$ with initial condition $y(0) = -1$.

7. Determine whether the following sequences $\{a_n\}_1^\infty$ converge or diverge. In the case of convergence, find the limit.

(3) (a) $\left\{\arccos\left(\dfrac{-5n^2}{5n^2+4}\right)\right\}$

(2) (b) $\left\{\left(\dfrac{n^n}{n \cdot n!}\right)\right\}$

8. Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answer.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n n!}{(2n+3)!}$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n+5}}$

(3) (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(2n)}{2^n+n}$

(4) **9.** Given that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, determine if $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n^2+2}$ converges or diverges. Justify your answer.

(5) **10.** Find the radius and interval of convergence for the following series,
$\displaystyle\sum_{n=1}^{\infty} \frac{6^n(4x-1)^{n-1}}{n}$

Answers Fall 2020

1. (a) $\ln|x-1| - \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan(x/2) + C$

(b) $3\arcsin\left(\dfrac{x-3}{3}\right) - \sqrt{6x-x^2} + C$

(c) $\dfrac{\sqrt{3}}{3}\pi - \ln 2$

(d) $\dfrac{\arcsin^2(x)}{2} + C$

2. (a) 1

(b) 1

3. Convergent; 2

4. $\displaystyle\int_1^3 ((3-x) - (3/x-1))\,dx$

5. (a) $\displaystyle\int_0^2 2\pi x e^{-x}\,dx$

(b) $\displaystyle\int_0^2 (\pi(3)^2 - \pi(3-e^{-x})^2)\,dx$

6. $-\sqrt{\dfrac{1}{3-2\sqrt{1+x^2}}}$

7. (a) Convergent; π

(b) Divergent

8. (a) Absolutely convergent

(b) Conditionally convergent

(c) Absolutely convergent

9. Diverges by test for divergence.

10. Radius $= 1/24$; Interval $5/24 \le x < 7/24$