Question 1. — Use polynomial long division to divide:

$$(50x^3 - 38x) \div (10x + 6)$$

(Express your answer in the form $Q(x) + \frac{R(x)}{D(x)}$.)

Question 2. — Solve the inequality for x: $x(x+1)(2x-1)^2 > 0$.

(Express your answer in interval notation.)

Question 3. — Given the quadratic function: $f(x) = -2x^2 - 2x + 4$:

a. Find the coordinates of all axis intercepts.

b. Find the coordinates of the vertex.

c. Sketch a graph of the function using the information from the previous parts.

Question 4. — Solve the equation for x:

$$\frac{-8}{x^2 + 2x - 15} + \frac{1}{x-3} = \frac{6}{x^2 + 5x}.$$

Question 5. — Given $f(x) = \dfrac{2x+7}{x-9}$, find $f^{-1}(x)$.

Question 6. — Solve the equation for x: $\sqrt{x^2+3} + 3 = x$.

Question 7. — Simplify the following expression as much as possible. Assume all variables are positive. Your final answer should not have any negative exponents.

$$\frac{9^{-\frac{1}{2}} (x^5 y^9)^{-\frac{1}{3}}}{\sqrt{36 x^{\frac{2}{3}} y^6}}$$

Question 8. — Let $f(x) = 4 - 2^{x+1}$.

a. Identify all intercepts and asymptotes.

b. Sketch the graph $y = f(x)$.

c. State the domain and range of $f(x)$.

Question 9. — $5000 is invested today at a 2.4% yearly interest rate. What will be the value of the investment 10 years from now if the interest is compounded quarterly? Give your answers to the nearest cent.

Question 10. — Evaluate the following expression in simplified exact form, without using decimals:

$$\ln\left(\frac{1}{\sqrt[7]{e^2}}\right).$$

Question 11. — Express as a single logarithm and simplify:

$$\tfrac{1}{2}\ln(xy) - \frac{3}{2}\ln(yz) - \frac{5}{2}\ln(xz).$$

Question 12. — Solve for x: $\log(x+5) + \log(x+1) = \log(x^2 + 2x + 21)$.

Question 13. — Solve for x: $3^{4x} = 7^{2-x}$.

Express your answer in the form $x = \frac{\ln(A)}{\ln(B)}$.

Question 14. — Given that θ is an acute angle and $\sin(\theta) = \frac{5}{13}$, find the exact values of the five remaining trigonometric functions.

Question 15. — If $\cos(\theta) = \frac{2}{3}$ and $\cot(\theta) < 0$, give the exact values of $\sin(\theta)$ and $\tan(\theta)$.

Question 16. — Find the exact values of all angles θ in the interval $[0, 2\pi)$ such that $\tan\theta = -\sqrt{3}$.

Question 17. — Prove the identity: $\cot x = \dfrac{\sin x}{\sec x - \cos x}$.

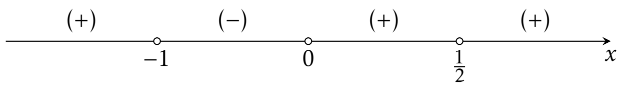
Question 18. — A triangle has sides of length $a = 5$, $b = 7$, and $c = 3$ across from angles of measure A, B, and C respectively. Which angle is the smallest? Find its measure accurate to two decimal places.

Question 19. — The angles of elevation to an airplane are measured from the top and the base of a building that is 20 m tall. The angle from the top of the building is 38° and the angle from the base of the building is 40°. Find the altitude of the airplane. Round your answer to the nearest metre.

Solution to question 1. — Cancelling the common factor before dividing gives

$$\frac{50x^3-38x}{10x+6}=\frac{25x^3-19x}{5x+3}=5x^2-3x-2+\frac{6}{5x+3}.$$

Solution to question 2. — The sign of the left side (which changes at -1 and 0, but not at $\frac{1}{2}$) is displayed below.

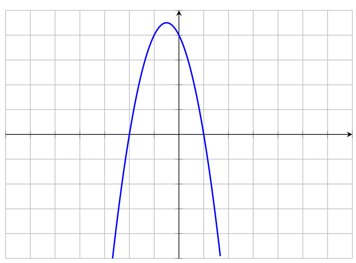


So the solution of the inequality is $(-\infty,-1)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

Solution to question 3. — a. $f(x)=-2x^2-2x+4=-2(x^2+x-2)=-2(x+2)(x-1)$, so the y intercept is $(0,4)$ and the x intercepts are $(-2,0)$ and $(1,0)$.

b. The x coordinate of the vertex is $-\frac{1}{2}$ and its y coordinate is $\frac{9}{2}$.

c. The graph is sketched below.



Solution to question 4. — Clearing denominators gives $-8x+x^2+5x=6x-18$, *i.e.*, $x^2-9x+18=0$, or $(x-3)(x-6)=0$, so the only solution is 6 (the left side of the equation is undefined if x is 3).

Solution to question 5. — The equation $y=\frac{2x+7}{x-9}$ is equivalent to $x\neq 9$ and $xy-9y=2x+7$, *i.e.*, $x(y-2)=9y+7$, or $x=\frac{9y+7}{y-2}$. Therefore, $f^{-1}(x)=\frac{9x+7}{x-2}$.

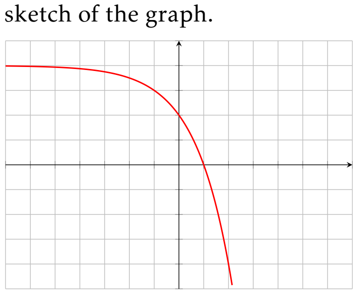
Solution to question 6. — The equation is equivalent to $\sqrt{x^2+3}-x=-3$, in which the left side is $\geqslant\sqrt{x^2}-x=|x|-x\geqslant 0$, so the equation has no solution.

Solution to question 7. — Expanding gives

$$\tfrac{1}{3}x^{-5/3}y^{-3}\cdot\tfrac{1}{6}x^{-1/3}y^{-3}=\tfrac{1}{18}x^{-2}y^{-6}=\frac{1}{18x^2y^6}.$$

Solution to question 8. — a. $f(0)=4-2=0$ and $f(x)=0$ is equivalent to $2^{x+1}=4$, *i.e.*, $x+1=2$ or $x=1$, so the x intercept is $(1,0)$ and the y intercept is $(0,2)$. The horizontal asymptote is defined by $y=4$.

b. Below is a sketch of the graph.



c. The domain of f is $\mathbb{R}$, and the range of f is $(-\infty,4)$.

Solution to question 9. — The multiplying factor for each compounding is $\frac{2.4}{100}\cdot\frac{1}{4}=\frac{3}{500}$, so after 10 years (= 40 compoundings) the values of the investment will be

$$\$5000\left(\tfrac{503}{500}\right)^{40}\approx\$6351.69.$$

Solution to question 10. — $\ln\left(\frac{1}{\sqrt[7]{e^2}}\right)=\ln\left(e^{-2/7}\right)=-\frac{2}{7}$.

Solution to question 11. — Combining the logarithms gives

$$\ln\left(x^{1/2}y^{1/2}y^{-3/2}z^{-3/2}x^{-5/2}z^{-5/2}\right)=\ln\left(\frac{1}{x^2yz^4}\right).$$

Solution to question 12. — Combining the logarithms gives

$$\log(x^2+6x+5)=\log(x^2+2x+21)$$

or equivalently, $x^2+6x+5=x^2+2x+21$, provided $x>-1$. Thus $4x=16$, so $x=4$.

Solution to question 13. — Applying the logarithm gives

$$4x\ln(3)=(2-x)\ln(7),\qquad\text{or}\qquad\big(4\ln(3)+\ln(7)\big)x=2\ln(7),$$

so $x=\dfrac{2\ln(7)}{4\ln(3)+\ln(7)}=\dfrac{\ln(49)}{\ln(81\cdot 7)}=\dfrac{\ln(49)}{\ln(567)}$.

Solution to question 14. — $\cos(\vartheta)=\sqrt{1-(5/13)^2}=\frac{12}{13}$, $\tan(\vartheta)=\frac{5}{12}$, $\sec(\vartheta)=\frac{13}{12}$, $\csc(\vartheta)=\frac{13}{5}$ and $\cot(\vartheta)=\frac{12}{5}$.

Solution to question 15. — If $\cos(\vartheta)>0$ and $\cos(\vartheta)<0$ then $\sin(\vartheta)=-\sqrt{1-(2/3)^2}=-\frac{1}{3}\sqrt{5}$, since $\sin(\vartheta)<0$, and $\tan(\vartheta)=-\frac{1}{2}\sqrt{5}$.

Solution to question 16. — If $\tan\vartheta=-\sqrt{3}$ and $0\leqslant\vartheta<2\pi$ then $\vartheta=\frac{2}{3}\pi$ or $\vartheta=\frac{2}{3}\pi+\pi=\frac{5}{3}\pi$.

Solution to question 17. — The right side of the given equation is

$$\frac{\sin x}{\sec x-\cos x}\cdot\frac{\cos x}{\cos x}=\frac{\sin x\cos x}{1-\cos^2 x}=\frac{\sin x\cos x}{\sin^2 x}=\frac{\cos x}{\sin x}$$

which is equal to the left side of the given equation.

Solution to question 18. — The smallest angle of $\triangle ABC$ is $\angle C$, since it is opposite the shortest side, and the law of cosines gives

$$\cos(\angle C)=\frac{5^2+7^2-3^2}{2\cdot 5\cdot 7}=\frac{65}{70}=\frac{13}{14},$$

so $\angle C=\cos^{-1}\left(\frac{13}{14}\right)\approx 21.79°$.

Solution to question 19. — If y is the altitude of the airplane and x is the horizontal distance from the building the point on the ground beneath the airplane, then

$$y=x\tan(40°)\qquad\text{and}\qquad y-20=x\tan(38°).$$

Multiplying the first by $\tan(38°)$, multiplying the second by $\tan(40°)$ and subtracting gives

$$y\big(\tan(40°)-\tan(38°)\big)=20\tan(40°),$$

or

$$y=\frac{20\tan(40°)}{\tan(40°)-\tan(38°)}\approx 290\,\text{m}.$$