(16) 1. Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

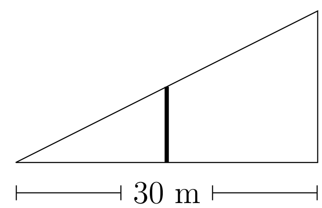
(a) $y = \dfrac{4}{5\sqrt[3]{x^2}} - 3^x + 2\ln(12x) - \log_5 4$

(b) $y = \dfrac{\tan^4(2x) - e^{3x}}{\sec(x^3)}$

(c) $\cos(2x) - 2xy^3 = \cot x + \sqrt{y}$

(d) $y = (\sin x - 2e^x)^{\csc x}$

(5) 2. A spotlight on the ground is shining on a wall 30m away. If a child who is 1 m tall runs from the spotlight toward the building at a speed of 2 m/s, how fast is the length of the child's shadow on the building decreasing when the child is 8 m from the building?



(10) 3. Consider the following function, along with its first and second derivatives.

$f(x) = \dfrac{3x^2 - 12}{x^2 + 12}$, $f'(x) = \dfrac{96x}{(x^2 + 12)^2}$, $f''(x) = \dfrac{-288(x^2 - 4)}{(x^2 + 12)^3}$

(a) Find the domain and intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals of increase/decrease of f.

(d) Find the local (relative) extrema of f.

(e) Find the intervals of concavity of f.

(f) Find all points of inflection of f.

(g) On the next page, sketch the graph of f.

(4) 4. Find the absolute maximum and absolute minimum of $f(x) = \sqrt[3]{x - x^2}$ on the interval $[-1, 2]$.

(5) 5. A box with an open top has vertical sides, a square bottom, and a volume of 32 cubic meters. If the box has the least possible surface area, find its dimensions.

(5) 6. Consider the definite integral $\displaystyle\int_0^4 (x^2 - 2x)\,dx$.

(a) Express it as a limit of a Riemann sum.

(b) Evaluate the limit to find the definite integral.

Note that: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(3) 7. Evaluate the integral by interpreting it in terms of areas.

$$\int_{-4}^{0} (1 + \sqrt{16 - x^2})\,dx$$

(9) 8. Evaluate the following integrals.

(a) $\displaystyle\int (7\sqrt[3]{x} - \frac{7}{x} + 5^x)\, dx$

(b) $\displaystyle\int \left(\frac{\tan^2 x + \sin^2 x + 2}{\sin^2 x}\right)\, dx$

(c) $\displaystyle\int_1^4 \frac{(x-1)^2}{\sqrt{x}}\, dx$

(3) 9. Let $f(x) = \displaystyle\int_1^{\cos x} \sqrt{e^t + 1}\, dt$.

(a) Compute $f(2\pi)$.

(b) Find $f'(x)$.

(3) 10. Given that

$$\int_1^6 f(x)\, dx = 8 \quad \text{and} \quad \int_4^6 f(x)\, dx = 3.$$

Find $\displaystyle\int_1^4 (3f(x) + 2)\, dx$.

(2) 11. Use differentiation to verify that

$$\int \tan x \sec^3 x\, dx = \frac{1}{3}\sec^3 x + C$$

Answers

1. (a) $\dfrac{dy}{dx} = \dfrac{-8}{15}\, x^{-5/3} - 3^x \ln 3 + \dfrac{2}{x}$

   (b) $\dfrac{dy}{dx} = \dfrac{(8\tan^3(2x)\sec^2(2x) - 3e^{3x})\sec(x^3) - (\tan^4(2x) - e^{3x})(3x^2\sec(x^3)\tan(x^3))}{\sec^2(x^3)}$

   (c) $\dfrac{dy}{dx} = \dfrac{\csc^2 x - 2\sin(2x) - 2y^3}{6xy^2 + \frac{1}{2\sqrt{y}}}$

   (d) $\dfrac{dy}{dx} = (\sin x - 2e^x)^{\csc x}\left(-\csc x \cot x \ln(\sin x - 2e^x) + \dfrac{\csc x(\cos x - 2e^x)}{\sin x - 2e^x}\right)$

2. Let y be the length of the shadow on the building. Then $\dfrac{dy}{dt} = -\dfrac{15}{121}$ m/s

3. (a) Domain: $(-\infty, \infty)$
   x-intercepts: $(\pm 2, 0)$
   y-intercept: $(0, -1)$

   (b) Vertical Asymptote: None
   Horizontal Asymptote: $y = 3$

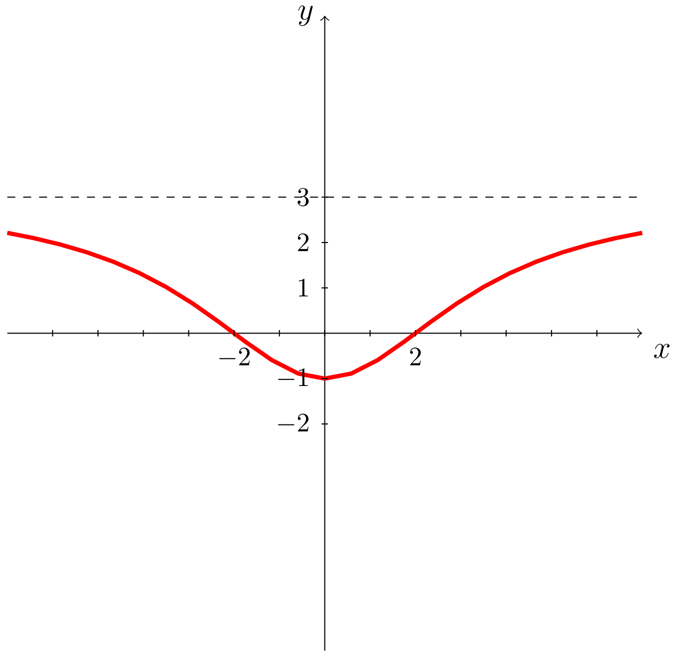
   (c) Decreasing on $(-\infty, 0)$
   Increasing on $(0, \infty)$

   (d) Local Minimum at $(0, -1)$
   Local Maximum : None

(e) Concave Upward on $(-2, 2)$
Concave Downward on $(-\infty, -2) \cup (2, \infty)$

(f) Points of Inflection: $(\pm 2, 0)$

(g) The graph



4. Absolute Minima at $(-1, -\sqrt[3]{2})$ and at $(2, -\sqrt[3]{2})$
   Absolute Maximum at $(1/2, \sqrt[3]{1/4})$

5. Bottom= 4 m by 4 m, Height= 2 m

6. (a) $\lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{4i}{n} \right)^2 - \frac{8i}{n} \right) \frac{4}{n} = \lim_{n\to\infty} \left(\frac{32(n+1)(2n+1)}{3n^2} - \frac{16(n+1)}{n} \right)$

   (b) $\frac{16}{3}$

7. $4 + 4\pi$

8. (a) $\frac{21}{4} x^{4/3} - 7 \ln |x| + \frac{5^x}{\ln 5} + C$

   (b) $\tan x + x - 2 \cot x + C$

   (c) $\frac{76}{15}$

9. (a) $f(2\pi) = 0$

   (b) $f'(x) = -\sqrt{e^{\cos x} + 1}(\sin x)$

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11. Note that $\left(\frac{1}{3} \sec^3 x + C \right)' = \cdots = \sec^3 x \tan x$