1. (10 points) Evaluate each improper integral, or show it diverges.

(a) $\displaystyle\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx$

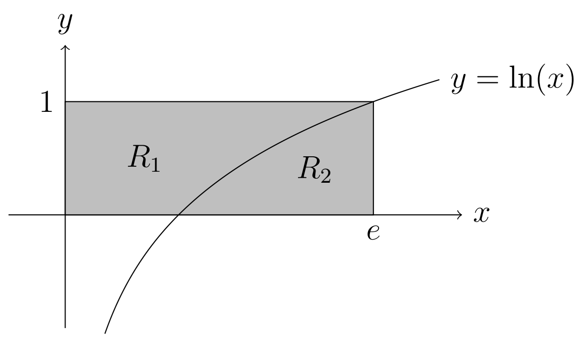
(b) $\displaystyle\int_0^1 \frac{\ln(x)}{\sqrt{x}}dx$

2. (5 points) Find the area of the region bounded by the curves $y = 4 - 3x - x^2$ and $y = 2 - 2x$.

3. (5 points) In the figure a shaded rectangle is divided into two regions, R_1 and R_2, by the curve $y = \ln(x)$. Write down, but do not evaluate, an integral for the volume of the solid obtained by

(a) rotating R_1 about the y-axis

(b) rotating R_2 about the line $y = -2$.



4. (4 points) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{3-y}{\sqrt{1-x^2}}$ given $y(0) = 5$.

5. (5 points) Determine whether the following sequences converge or diverge. In the case of convergence, find the limit.

(a) $\left\{\left(\dfrac{n+2}{n}\right)^n\right\}$

(b) $\left\{\dfrac{(2n+1)!}{n^2(2n-1)!}\right\}$

6. (4 points) Consider the series given by $\displaystyle\sum_{m=1}^{\infty} \frac{1}{(m+2)(m+3)}$.

(a) Find a simple formula for the sequence of partial sums of this series.

(b) Does the series converge or diverge? If it converges find its sum.

7. (9 points) Determine whether the following series converge or diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+2}{2^{2n}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{e^k}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

8. (8 points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2+n+1}$

9. (5 points) Determine the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n\sqrt{n+1}}(x-2)^n$.

10. (5 points) Find the Taylor series for $f(x) = \dfrac{1}{x-1}$ centered at 3. Write the first four terms of the series explicitly, and express the series using appropriate sigma notation.

Answers

1. (a) $\pi/4$
   (b) -4
2. $9/2$
3. (a) $\displaystyle\int_0^1 \pi e^{2y}\,dy$
   (b) $\displaystyle\int_1^e \pi\left[(\ln x + 2)^2 - 4\right]dx$
4. $y = 3 + 2e^{-\arcsin(x)}$
5. (a) e^2
   (b) 4
6. (a) $s_n = \dfrac{1}{3} - \dfrac{1}{n+3}$
   (b) The series converges and its sum is 1/3.
7. (a) Converges (*Series can be written as a sum of two converging geometric series.*)
   (b) Converges (*Result can be obtained by the Ratio Test.*)
   (c) Diverges (*Result can be obtained by the Integral Test.*)
8. (a) Conditionally convergent (*Series converges by the Alternating Series Test. Corresponding series diverges by the Limit Comparison Test with* $\sum \frac{1}{n^{1/2}}$.)
   (b) Absolutely convergent (*Corresponding series converges by the Comparison Test with* $\sum \frac{1}{n^2}$.)
9. $(-3, 7]$
10. $\dfrac{1}{2} - \dfrac{1}{2^2}(x-3) + \dfrac{1}{2^3}(x-3)^2 - \dfrac{1}{2^4}(x-3)^3 + \cdots$
    $= \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-3)^n$