1. (30 points) Evaluate the following integrals.

(a) $\displaystyle\int \left[\frac{x^2\tan(x) + \sqrt[3]{x^7}}{x^2} + (x+1)(2x)\right] dx$

(b) $\displaystyle\int \frac{18}{(x-1)(x+2)^2} dx$

(c) $\displaystyle\int_0^5 |3x-6|\, dx$

(d) $\displaystyle\int 2x^5 \sin(x^2) dx$

(e) $\displaystyle\int_1^8 \frac{e^{\sqrt{3x+1}}}{\sqrt{3x+1}} dx$

(f) $\displaystyle\int_1^e x^3 \ln(x^2) dx$

2. (4 points) Find $f(x)$ given that $f''(x) = 84x^5 - 6$, $f'(0) = 4$, and $f(1) = 0$.

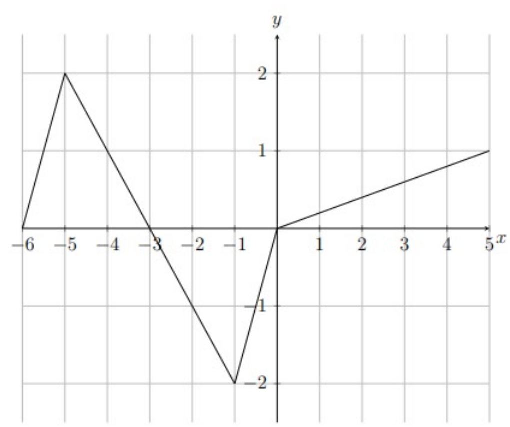
3. (4 points) Approximate $\displaystyle\int_0^2 \sqrt{16 - e^x}$ using the Right-End Point method with n=4. Round your answer to 4 decimals.

4. (3 points) The function $f(x)$ is given by the graph below.

Find

(a) $\displaystyle\int_{-6}^{5} f(x)\, dx$

(b) $\displaystyle\int_{-5}^{-5} f(x)\, dx$



5. (5 points) Find the area of the region enclosed by the curves $y = 2x^2 + 2x$ and $y = x^2 + 3$.

6. (5 points) The demand function of a product is $p = \frac{160}{\sqrt{1+3x}}$, and the supply function is $p = 10\sqrt{1+3x}$.

(a) Find the equilibrium point.

(b) Produce a rough sketch and label the region whose area represents the producer surplus.

(c) Using the quantities present in this scenario, set up, but do not evaluate, the integral for producer surplus.

7. (5 points) Find the function y that satisfies the differential equation $y' = 2xy - 3x$, with $y(0) = -1$ and $y < 0$.

8. (4 points) The area A (in km^2) of the range of an invasive species is expanding at a rate proportional to the time t (in years) since the species was introduced. We can assume the area was 0 km^2 when $t = 0$. The area of the range 2 years later is 1266 km^2.

(a) Write the differential equation.

(b) Find the equation for $A(t)$.

(c) When will the area of the range be 16000 km^2?

9. (6 points) Evaluate the limits.

(a) $\displaystyle\lim_{x\to\pi} \frac{1+\cos x}{(x-\pi)^2}$

(b) $\displaystyle\lim_{x\to-\infty} \frac{x^2+e^x+7}{x^3+e^x-3}$

10. (8 points) Evaluate the improper integrals.

(a) $\displaystyle\int_{-\infty}^{0} \frac{1}{(1-x)^{\frac{3}{2}}} dx$

(b) $\displaystyle\int_{2}^{4} \frac{x}{x^2-4} dx$

11. (2 points) Find a formula for the n^{th} term of the sequence $\left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{4}, \frac{5}{8}, \frac{6}{16}, \ldots\right\}$

12. (4 points) Does the sequence converge or diverge? If it converges, find the limit.

(a) $\left\{\dfrac{\sqrt{16n^2+5}}{10n-3}\right\}$

(b) $\left\{\dfrac{5\cdot 8^n}{2^{3n+1}}\right\}$

13. (16 points) Determine whether the following series converge or diverge. Identify which test you are using. In case of a convergent geometric or telescoping series, find the sum of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \left[5n^{-5} + \frac{1}{\sqrt{n^3}}\right]$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}n}{(n-1)!}$

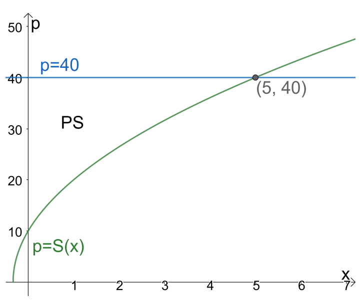
(c) $\displaystyle\sum_{n=1}^{\infty} \left[\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}\right]$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{3}{8}\left(-\frac{2}{3}\right)^{n+1}$

(e) $\displaystyle\sum_{n=0}^{\infty} \cos\left(\frac{1}{n+1}\right)$

14. (4 points) \$9 is invested twice per year in high interest savings account giving 6% annual interest compounded twice per year. What is the balance after 30 years?

ANSWERS

1. a)$-\ln|\cos(x)| + \frac{3}{4}x^{\frac{4}{3}} + \frac{2}{3}x^3 + x^2 + C$
b)$2\ln|x-1| - 2\ln|x+2| + \dfrac{6}{x+2} + C$
c)$\frac{39}{2}$
d)$-x^4\cos(x^2) + 2x^3\sin(x^2) + 2\cos(x^2) + C$
e)$\frac{2}{3}(e^5 - e^2) + C$
f)$\frac{3}{8}e^4 + \frac{1}{8}$
2. $2x^7 - 3x^2 + 4x - 3$
3. 6.8805
4. a)$\frac{5}{2}$b)0
5. $\frac{32}{3}$
6. a)$(5, 40)$ b) 

c)$\int_0^5 (40 - 10\sqrt{1+3x})dx$
7. $y = \frac{-5}{2}e^{x^2} + \frac{3}{2}$
8. a)$\frac{dA}{dt} = kt$ b)$A = \frac{633}{2}t^2$ c)About 7.1 years
9.a)$\frac{1}{2}$ b)0
10. a)converges to 2 b)diverges to ∞
11. $a_n = \frac{n+1}{2^{n-1}}$
12. a) converges to $\frac{2}{5}$ b)converges to $\frac{5}{2}$
13.a)converges (p-series and properties of convergent series)
b) converges (ratio test) c) converges to $1 + \frac{1}{\sqrt{2}}$ (telescoping)
d)converges to $-\frac{1}{15}$ (geometric) e) diverges (test for divergence)
14. \$ 1511.51