1. (9 points) Evaluate the following limits. Use $-\infty$, ∞, or "does not exist," wherever appropriate.

(a) $\displaystyle\lim_{x\to 3} \frac{x^3 - x^2 - 6x}{x^2 + 2x - 15}$

(b) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+12} - 4}{x - 4}$

(c) $\displaystyle\lim_{x\to 0^-} \frac{\sin x}{x^2}$

2. (6 points) Find all the vertical and horizontal asymptotes for $f(x) = \dfrac{3x + \sqrt{x^2+1}}{2x - 1}$.

3. (5 points) Given $f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x^2 + 2 & \text{if } -1 < x < 3 \\ 2^x + 2 & \text{if } x \geq 3 \end{cases}$

Determine all points of discontinuity of f. Identify the type of any discontinuity you find (removable, jump or infinite). Fully justify your answer.

4. (5 points) Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{2 - x}$.

5. (15 points) Find $\frac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = x^2 + 2^x + \ln x - \dfrac{1}{2x} + \sqrt[3]{x^2} + \sqrt{e^{2\pi}}$

(b) $y = \sec^5\left(\dfrac{x+1}{x-1}\right)$

(c) $y = (1 + 2x)^{\sin x}$

6. (5 points) Let $f(x) = (x - 3)[h(x^2)]^2$. Given that $h(4) = 10$ and $h'(4) = 3$, find $f'(2)$.

7. (6 points) Use implicit differentiation to find an equation of the tangent line to the curve $x^4 + x^2y + y^5 = 11$ at the point $(2, -1)$.

8. (7 points) At noon, ship A is 30 km west of ship B. Ship A heads west at a constant speed of 10 km/h. Ship B heads north at a varying speed. At 3pm, ship B has travelled 80 km and is currently going 50 km/h. How fast is the distance between the ships changing at that moment?

9. (6 points) Find the absolute maximum and absolute minimum of $f(x) = \cos x + x \sin x$ on the interval $[0, \pi]$.

10. (12 points) Consider the following function, along with its first and second derivatives.

$$f(x) = \frac{27(x-2)}{x^3} \qquad f'(x) = \frac{54(3-x)}{x^4} \qquad f''(x) = \frac{162(x-4)}{x^5}$$

(a) Find the domain and any intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals of increase/decrease of f.

 Question 10 continues on the next page.

(d) Find the local (relative) extrema of f.

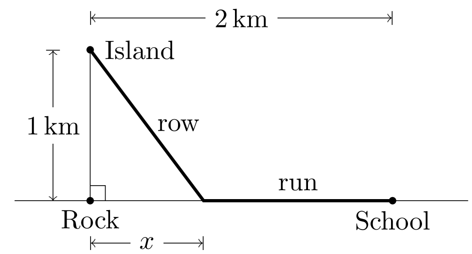
(e) Find the intervals of upward and downward concavity of f.

(f) Find all points of inflection of f.

(g) On the next page, sketch a graph of f.

11. (7 points) A student lives on an island, 1 km away from the mainland. The shore of the mainland is completely straight. There is a large rock at the point on the shoreline closest to the island. The student's school is on the shore of the mainland, 2 km away from the rock. The student can row at 5 km/h, and can run at 13 km/h. How far from the rock should the student land their rowboat in order to minimize the time it takes them to get to school? (Suggestion: let x be the distance from the rock to the point where they land, as shown in the given diagram.)

Hint: Remember that $\textit{time} = \dfrac{\textit{distance}}{\textit{speed}}$.



12. (4 points) Compute $\int_0^2 (2x^3 - 1)\,dx$ as a limit of Riemann sums. Note that $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$.

13. (8 points) Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{(2x+3)(x^2+1)}{x}\,dx$

(b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\sin\theta + \frac{1}{\sin^2\theta}\right) d\theta$

14. Consider the function $g(x) = \displaystyle\int_{x^2}^{\pi} \frac{e^t}{t}\,dt$.

(a) (1 point) Evaluate $g(\sqrt{\pi})$.

(b) (3 points) Using the Fundamental Theorem of Calculus, find $g'(x)$.

(c) (1 point) Find the slope of the tangent line to $y = g(x)$ when $x = 1$.

 Question 14 continues on the next page.

Answers:

1. a) $\dfrac{15}{8}$ b) $\dfrac{1}{8}$ c) $-\infty$

2. VA: $x = \dfrac{1}{2}$ HA: $y = 1$ and $y = 2$

3. Removable discontinuity when $x = -1$, since $\lim_{x\to -1^-} f(x) = 3$ and $\lim_{x\to -1^+} f(x) = 3$ but $f(-1)$ is undefined. Jump discontinuity when $x = 3$, since $\lim_{x\to 3^-} f(x) = 11$ and $\lim_{x\to 3^+} f(x) = 10$.

4. $\dfrac{1}{(2-x)^2}$

5. a) $2x + 2^x \ln 2 + \dfrac{1}{x} + \dfrac{1}{2x^2} + \dfrac{2}{3x^{1/3}}$ b) $y' = -5\sec^5\left(\dfrac{x+1}{x-1}\right) \cdot \tan\left(\dfrac{x+1}{x-1}\right) \cdot \dfrac{2}{(x-1)^2}$
c) $y' = (1+2x)^{\sin x}\left(\cos x \, \ln(1+2x) + \dfrac{2\sin x}{1+2x}\right)$

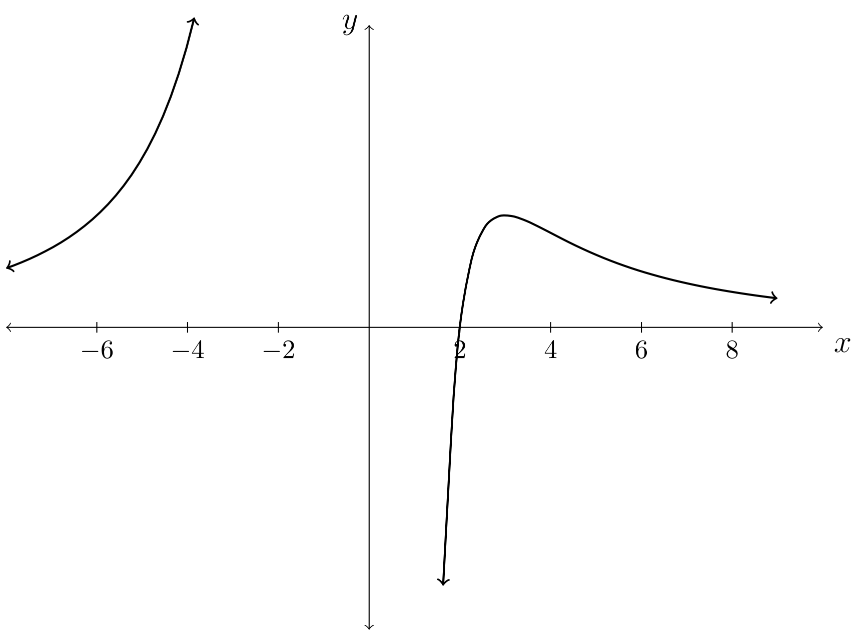
6. -140

7. $y = -\frac{28}{9}x + \frac{47}{9}$

8. 46 km/h

9. Maximum $\frac{\pi}{2}$ (attained at $x = \frac{\pi}{2}$); minimum -1 (attained at $x = \pi$).

10. Domain $x \in (\infty, 0) \cup (0, \infty)$; x-int at $(2, 0)$; VA $x = 0$, HA $y = 0$; increasing for $x < 0$ and $0 < x < 3$, and decreasing for $x > 3$; local max at $(3, 1)$; concave upward for $x < 0$ and $x > 4$, and concave downward for $0 < x < 4$; point of inflection at $(4, \frac{27}{32})$.



 Question 14 continues on the next page.

11. $\frac{5}{12}$ km

12. 6

13. a) $\frac{2}{3}x^3 + \frac{3}{2}x^2 + 2x + 3\ln|x| + C$ b) $\frac{1}{2} + \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{3}}$

14. a) 0 b) $\dfrac{-2e^{x^2}}{x}$ c) $-2e$