

1. Evaluate the following integrals.

(6) (a) $\int \frac{x^2 - 8x - 21}{(x - 3)(x^2 + 9)} dx$

(6) (b) $\int_{\pi/4}^{\pi/2} \csc^3 x \cot x dx$

(6) (c) $\int \frac{x^2}{\sqrt{16 - x^2}} dx$

(6) (d) $\int \frac{e^x}{e^{2x} - 4} dx$

(6) (e) $\int_0^1 x \arctan(x^2) dx$

2. Evaluate the following limits.

(3) (a) $\lim_{x \rightarrow \pi/2} \frac{\cos^2(5x)}{1 + \sin(3x)}$

(3) (b) $\lim_{x \rightarrow \pi^-} (x - \pi) \csc x$

(3) (c) $\lim_{x \rightarrow 0^+} (1 + 5x)^{\frac{1}{2x}}$

(8) 3. Determine whether the following improper integrals converge or diverge. If an integral converges, give its exact value.

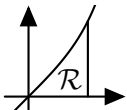
(a) $\int_0^1 \frac{1}{2 - 3x} dx$

(b) $\int_0^{\infty} \sin \theta e^{-\cos \theta} d\theta$

(4) 4. Solve the differential equation: $(1 + \cos x) e^y \frac{dy}{dx} = (e^y + 1) \sin x$, with initial condition $y(0) = 0$.

(5) 5. The number $N(t)$ of people in a community who are exposed to a particular advertisement is governed by the logistic equation. Initially $N(0) = 50$, and it is observed that $N(1) = 200$. Solve for $N(t)$ if it is predicted that the maximum number of people in the community who will see the advertisement is 300.

(6) 6. Given \mathcal{R} , the region bounded by $y = \tan(x)$, $x = \pi/4$ and $y = 0$.



Set up, **but do not evaluate**, the integrals needed to find the volume of the solids of revolution obtained by revolving \mathcal{R} about:

(a) the x -axis.

(b) the line $x = -2$.

(c) the line $y = 3$.

(4) 7. (a) Does the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n-1)!}$ converge? Justify your answer.

- (b) Does the corresponding sequence $\left\{ \frac{2^n}{(2n-1)!} \right\}$ converge? Justify your answer.
8. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.
- (4) (a) $\sum_{n=1}^{\infty} \frac{n \sin^2 n}{1+n^3}$
- (4) (b) $\sum_{n=1}^{\infty} \ln \left(\frac{n}{3n+1} \right)$
- (4) (c) $\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$
- (4) 9. Find the sum of the series $\sum_{n=1}^{\infty} \left(\cos \left(\frac{1}{n+1} \right) - \cos \left(\frac{1}{n} \right) \right)$.
10. Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answer.
- (4) (a) $\sum_{n=1}^{\infty} (-1)^n \arctan(n+1)$
- (4) (b) $\sum_{n=1}^{\infty} \frac{(-4)^{2n+1}}{n^2 7^n}$
- (5) 11. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-2)^n (x-2)^n}{\sqrt{n+3}}$.
- (5) 12. Find the Maclaurin series expansion of $f(x) = \ln(4-x)$, and express the series using the appropriate sigma notation.

Answers:

1a. $-2 \ln|x-3| + 3/2 \ln(x^2+9) + 1/3 \arctan(x/3) + C$

1b. $-1/3 + 2\sqrt{2}/3$

1c. $8 \arcsin(x/4) - x\sqrt{16-x^2}/2 + C$

1d. $1/4 \ln|(e^x-2)/(e^x+2)| + C$

1e. $\pi/8 - (\ln 2)/2$

2a. $50/9$

2b. -1

2c. $e^{5/2}$

3a. ∞ , divergent

3b. DNE, divergent

4. $y = \ln|4/(1+\cos x) - 1|$

5. $N = 300 \cdot 10^t / (5 + 10^t)$

6a. $V = \pi \int_0^{\pi/4} \tan^2 x dx$

6b. $V = \pi \int_0^1 ((2 + \pi/4)^2 - (2 + \arctan y)) dy$

6c. $V = \pi \int_0^{\pi/4} (3^2 - (3 - \tan x)^2) dx$

7a. ratio test, AC

7b. conv. to 0

8a. comparison test, conv.

8b. nTT, divergent

8c. ratio test, AC

9. telesc. series, conv. to $1 - \cos 1$

10a. nTT, div.

10b. ratio test, div.

11. $I = (3/2, 5/2]$, $R = 1/2$

12. $f(x) = \ln 4 + \sum_{n=1}^{\infty} (-x^n / (4^n \cdot n))$