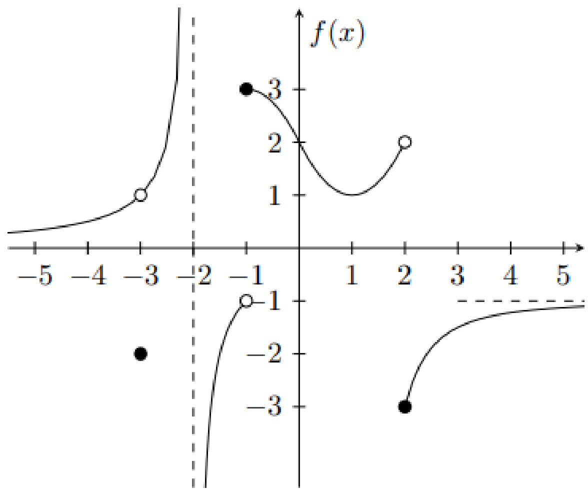
(6) 1. For the function f whose graph is shown below, determine each of the following. Use "does not exist" (DNE), ∞ or $-\infty$, where appropriate.



a) $\lim_{x\to\infty} f(x) =$

b) $\lim_{x\to-3} f(x) =$

c) $f(-3) =$

d) $\lim_{x\to-1^-} f(x) =$

e) $\lim_{x\to-1} f(x) =$

f) $f'(1) =$

g) $\lim_{x\to-2^+} f(x) =$

h) List all x-values, if any, where the function is discontinuous.

i) List all x-values, if any, where the function is continuous but not differentiable.

(20) 2. Evaluate the following limits. Use "does not exist" (DNE), ∞ or $-\infty$, where appropriate.

a) $\lim_{x\to-4} \frac{2x^2+7x-4}{x^2+x-12}$

b) $\lim_{x\to3} \frac{\frac{1}{2x-1}-\frac{1}{x+2}}{x-3}$

c) $\lim_{x\to2^+} \frac{|4-2x|}{x^2-4}$

d) $\lim_{x\to2^-} \frac{1-x^2}{2-x}$

e) $\lim_{x\to-\infty} \frac{-3x^3+4x}{(2-x)(6+x)^2}$

(5) 3. Use the definition of continuity to determine the points of discontinuity of the following function:

$$f(x)=\begin{cases}\dfrac{1}{x^2-4} & if\ \ x\le -4\\ \dfrac{x^2+5x+6}{x^2-x-12} & if -4<x\le 0\\ \dfrac{3}{x-6} & if\ x>0\end{cases}$$

(4) 4. Find the value(s) of the constant k for which the following function is continuous for all real numbers.

$$f(x)=\begin{cases}k+10x & if\ \ x<2\\ 3k^2-4kx-5x & if\ x\ge 2\end{cases}$$

(5) 5. Use the limit definition of the derivative to calculate the derivative of $f(x)=\sqrt{5x-2}$.

(20) 6. Compute $\frac{dy}{dx}$ for each of the following equations. Use properties of logarithms where appropriate. Do NOT simplify your answers.

a) $y=(x^2-\log_3 x+\pi^e)^{10}\sin^2 x$

b) $y=\frac{2^{x^2}-x}{\sec x}$

c) $y=\ln[(2x^2-1)^3e^{2x}]$

d) $y=\sqrt[3]{\left(x^3-e^{x^4}\right)^7}$

e) $y=(x^4-5)^{\tan x}$

(4) 7. Find the 4th derivative of $f(x)=\frac{x^4}{12}-7x^3+e^{2x}$.

(4) 8. Find the value(s) of x at which the tangent line to the graph of

$$f(x)=\sqrt[3]{x^2-8x}$$

is horizontal.

(5) 9. Find an equation of the tangent line to the curve $e^{xy}=x-y$ at the point $(1,0)$.

(5) 10. Find the absolute extrema of $f(x)=(3x-2)^3(x+5)^6$ on the interval $[-4,0]$.

(10) 11. Consider $f(x) = \frac{x^2}{3(x^2-9)}$, with $f'(x) = \frac{-6x}{(x^2-9)^2}$, and $f"(x) = \frac{18(x^2+3)}{(x^2-9)^3}$.

Determine the following, then neatly sketch the graph of $f(x)$.

a) the domain of f,
b) all vertical and horizontal asymptotes,
c) all $x-$ and $y-$ intercepts,
d) the intervals on which f is increasing and decreasing,
e) all local extrema of f,
f) the intervals on which f is concave up and concave down,
g) the inflection points of f,
h) sketch a graph of f and clearly label any important points.

(6) 12. Pianomania store sells Yahaha keyboards at $800 each. They sell on average 12 such keyboards a month. A marketing consultant suggests that reducing the price of the keyboard by $20 would increase sales by 2 keyboards a month.

a) Find the keyboard price that would maximize revenue per month.
b) What is the maximum revenue per month?

(6) 13. Suppose that the demand for the new Nemo floppy fish cat toy is given by the equation

$$x = 1250 - 2p^2,$$

where x is the quantity demanded (the number of cat toys).

a) Find the price elasticity of demand function $E(p)$.
b) When $p = \$15$, is demand elastic, inelastic, or unitary?
c) When $p = \$15$, if price is increased by 2%, how is demand affected?
d) Find the price that will maximize the revenue.

<u>Answers:</u>

1. a) -1 b) 1 c) -2 d) -1 e) DNE f) 0 g) $-\infty$ h) $-3, -2, -1, 2$ i) none
2. a) $\frac{9}{7}$ b) $-\frac{1}{25}$ c) $\frac{1}{2}$ d) $-\infty$ e) 3
3. Discontinuity at $x = -4, -3, 6$
4. $k = -2, 5$
5. $f'(x) = \frac{5}{2\sqrt{5x-2}}$

6. a) $y' = 10(x^2 - \log_3 x + \pi^e)^9 \left(2x - \frac{1}{x\ln 3}\right)\sin^2 x + 2\sin x\cos x\,(x^2 - \log_3 x + \pi^e)^{10}$

   b) $y' = \frac{\left(2^{x^2}\ln 2(2x) - 1\right)\sec x - \sec x\tan x\left(2^{x^2} - x\right)}{\sec^2 x}$

   c) $y' = \frac{12x}{2x^2-1} + 2$

   d) $y' = \frac{7}{3}\left(x^3 - e^{x^4}\right)^{\frac{4}{3}}\left(3x^2 - 4x^3e^{x^4}\right)$

   e) $y' = (x^4 - 5)^{\tan x}\left(\sec^2 x\ln(x^4 - 5) + \frac{4x^3\tan x}{x^4-5}\right)$

7. $f^{(4)}(x) = 2 + 16e^{2x}$
8. $x = 4$
9. $y = \frac{1}{2}x - \frac{1}{2}$
10. absolute maximum at $(-4, -2744)$, absolute minimum at $\left(-\frac{11}{9}, -528\,933.95\right)$
11. a) $\{x | x \neq -3, 3\}$

    b) V.A. $x = -3$ and $x = 3$, H.A. $y = \frac{1}{3}$

    c) $x -$ and $y -$ intercept $(0,0)$

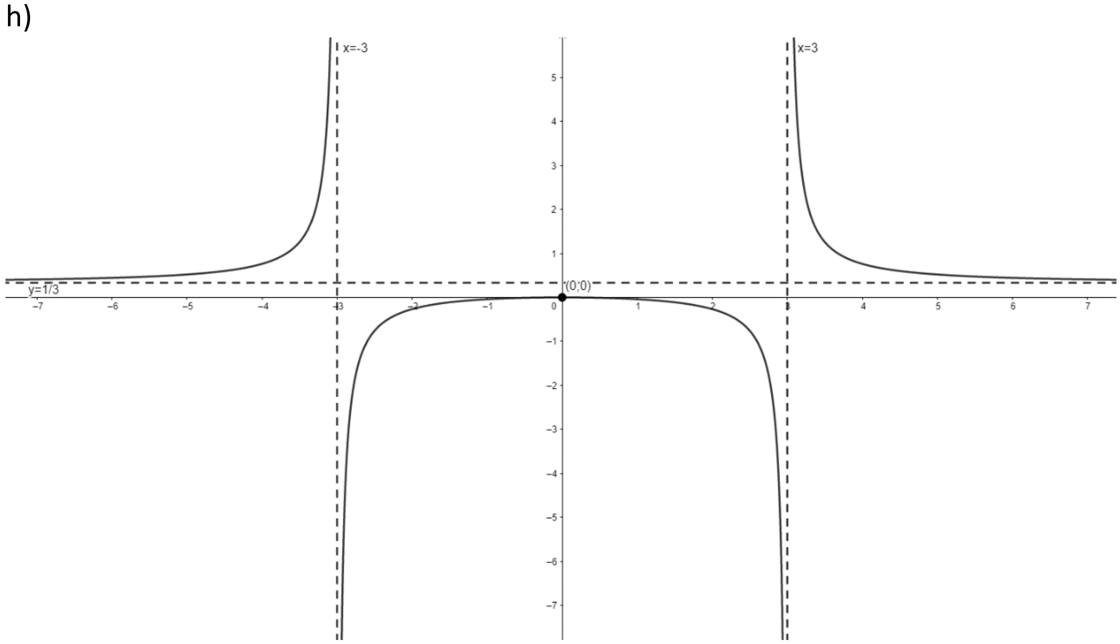
    d) increasing on $(-\infty, -3), (-3, 0)$, decreasing on $(0, 3), (3, \infty)$

    e) local maximum at $(0, 0)$

    f) concave up on $(-\infty, -3), (3, \infty)$, concave down on $(-3, 3)$

    g) none

    h)



12. a) \$460 b) \$21 160
13. a) $E(p) = \frac{2p^2}{625-p^2}$ b) elastic c) demand will decrease by 2.25% d) \$14.43