Logarithms Problem Set

Attempt the following problems *without using a calculator*.

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1. Convert the exponential equation to a logarithmic equation, or vice versa:

Ex.	$27^{2/3} = 9$	$\log_{27}(9) = \frac{2}{3}$
(a)	$4^{3/2} = 8$	
(b)	$4^x = \frac{1}{16}$	
(c)	$e^3 = y$	
(d)		$\ln(e) = 1$
(e)		$\log_a(3) = 6$
(f)	$e^x = 8$	
(g)		$\log_2(-5) = y$

2. Find the value of:

(a) $\log_7\left(\frac{1}{7}\right)$
(b) $\log_{\frac{1}{4}} 2$
(c) $\log_a(a)$
(d) $\ln(0)$
(e) $\ln(e^{x^2-4})$
(f) $5^{2\log_5(3)}$
(g) $\log_4 1$
(h) $\ln(1)$
(i) $\log_4(4)$
(j) $\ln(e)$
(k) $\ln(4e)$
(l) $\ln(e+4)$
(m) $e^{\ln(1)}$
(n) $e^{\ln(-6)}$
(o) $\log_4(-8)$
(p) $\log_2(4^x)$

3. Solve for x:

(a) $e^x = 9$
(b) $e^x = -1$
(c) $2^x = 0$
(d) $2^x = \frac{1}{9}$
(e) $\log_x(4) = 9$
(f) $3^{x-5} = 4^x$
(g) $e^{4x+3} = 2$
(h) $2e^{3x+5} = 7$
(i) $3e^{x+1} = 2e^{2x}$

4. Suppose the graph of $y = a^x$ has the following properties:

- Passes through the point $(2, 3)$
- On the left-hand side of the graph (when x-is a large negative number), the y-values are small positive numbers

(a) Sketch the graph of $\log_a(x)$.
(b) What is the value of a?

5. If $a^x = 3$, find the value of the following. (In some cases your answer will involve a).

(a) a^{3x}
(b) a^{x-1}
(c) a^{4x+1}
(d) $4a^{-\frac{x}{2}}$

6. Factor $e^{4x} - 4^{2x}$ as a difference of squares.

Solutions

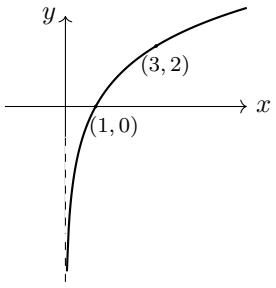
1.

Ex.	$27^{2/3} = 9$	$\log_{27}(9) = \frac{2}{3}$
(a)	$4^{3/2} = 8$	$\log_4(8) = \frac{3}{2}$
(b)	$4^x = \frac{1}{16}$	$\log_4\left(\frac{1}{16}\right) = x$
(c)	$e^3 = y$	$\ln(y) = 3$
(d)	$e^1 = e$	$\ln(e) = 1$
(e)	$a^6 = 3$	$\log_a(3) = 6$
(f)	$e^x = 8$	$ln(8) = x$
(g)	$2^y = -5$	$\log_2(-5) = y$

2. (a) -1
   (b) $-\frac{1}{2}$
   (c) 1
   (d) undefined (logarithms can only accept **positive** values)
   (e) $x^2 - 4$
   (f) 9
   (g) 0
   (h) 0
   (i) 1
   (j) 1
   (k) $\ln(4) + 1$
   (l) cannot be further simplified
   (m) 1
   (n) undefined ($e^{()}$ and $\ln()$ would cancel each other out, but $\ln(-6)$ is not defined in the first place)
   (o) undefined (logarithms can only accept **positive** values)
   (p) $2x$

3. (a) $\ln(9)$
   (b) no solution
   (c) no solution
   (d) $\log_2\left(\frac{1}{9}\right)$ or $-\log_2(9)$
   (e) $\sqrt[9]{4}$
   (f) $\frac{5\ln(3)}{\ln(3) - \ln(4)}$ or $\frac{5\ln(3)}{\ln\left(\frac{3}{4}\right)}$
   (g) $\frac{\ln(2) - 3}{4}$
   (h) $\frac{\ln\left(\frac{7}{2}\right) - 5}{3}$
   (i) $\ln\left(\frac{3}{2}\right) + 1$

4. (a)



   (b) $a = \sqrt{3}$

5. (a) $3^3 = \mathbf{27}$ (b) $3 \cdot a^{-1} = \mathbf{\frac{3}{a}}$ (c) $3^4 \cdot a = \mathbf{81a}$ (d) $4(3)^{-1/2} = \mathbf{\frac{4}{\sqrt{3}}}$

6. $e^{4x} - 4^{2x} = \left(e^{2x}\right)^2 - \left(4^x\right)^2 = \mathbf{\left(e^{2x} - 4^x\right)\left(e^{2x} + 4^x\right)}$