201-103-RE - Supplement B: One-Sided Limits and Continuity

Evaluate the following limits using ∞, $-\infty$ and "does not exist" where appropriate.

(1) $\displaystyle\lim_{x\to1^+}\frac{3-x}{x-1}$

(2) $\displaystyle\lim_{x\to3^-}\frac{6x-x^2}{x-3}$

(3) $\displaystyle\lim_{x\to2^+}\frac{4x}{2-x}$

(4) $\displaystyle\lim_{x\to2^-}\frac{x^2-1}{x-2}$

(5) $\displaystyle\lim_{x\to4^+}\frac{9-x^2}{x-4}$

(6) $\displaystyle\lim_{x\to3^-}\frac{2x}{2x-6}$

(7) $\displaystyle\lim_{x\to-1^+}\frac{x^2+2x}{x^2-1}$

(8) $\displaystyle\lim_{x\to2^-}\frac{3x+4}{x-2}$

(9) $\displaystyle\lim_{x\to3}\frac{2x^2-5x-3}{x^3-3x^2-4x+12}$

(10) $\displaystyle\lim_{x\to1^-}\frac{x^2-1}{x^2+x-2}$

(11) $\displaystyle\lim_{x\to3^-}\frac{\sqrt{3-x}}{x^2-9}$

Evaluate the following limits using ∞, $-\infty$ and "does not exist" where appropriate.

(12) $\displaystyle\lim_{x\to2^+}\frac{x^2+x-6}{|2-x|}$

(13) $\displaystyle\lim_{x\to3^-}\frac{|9-3x|}{x^2-9}$

(14) $\displaystyle\lim_{x\to-2^-}\frac{4x^2+9x+2}{|4-x^2|}$

For the following questions, find the discontinuities of the given function.

(15) $r(x)=\dfrac{x^2+2x-3}{x^2-1}$

(16) $q(x)=\begin{cases}4x-1 & \text{if } x\leqslant1 \\ 2-x^2 & \text{if } x>1\end{cases}$

(17) $f(x)=\begin{cases}\sqrt{x}+2 & \text{if } x\geqslant1 \\ 4-x+x^2 & \text{if } x<1\end{cases}$

(18) $g(x)=\dfrac{x+2}{x^2+x-2}$

(19) $t(x)=\dfrac{3x^2}{6x+x^2}$

(20) $g(x)=\dfrac{x^2-2x-3}{x^2-9}$

(21) $h(x)=\begin{cases}x^2-1 & \text{if } x\leqslant-2 \\ 3x+1 & \text{if } x>-2\end{cases}$

(22) $G(x)=\begin{cases}x^2+x & \text{if } x\leqslant-1 \\ x^3 & \text{if } x>-1\end{cases}$

(23) $h(x)=\dfrac{x-3}{x^2-3x}$

(24) $f(x)=\dfrac{x^2}{4x-x^2}$

(25) $h(x)=\dfrac{4-x}{x^2-7x+12}$

(26) $f(x)=\begin{cases}3x-5 & \text{if } x<2 \\ \sqrt{x-1} & \text{if } x>2\end{cases}$

(27) $f(x)=\begin{cases}5x-1 & \text{if } x>1 \\ 2 & \text{if } x=1 \\ 3x^2+1 & \text{if } x<1\end{cases}$

(28) $m(x)=\begin{cases}11-x^2 & \text{if } x\geqslant-3 \\ |x+1| & \text{if } x<-3\end{cases}$

(29) $s(x)=\dfrac{2-x}{x^2+2x-8}$

(30) $N(x)=\begin{cases}x^2+3 & \text{if } x<-2 \\ -2x+3 & \text{if } x>-2\end{cases}$

(31) $p(x)=\begin{cases}\sqrt{x+6} & \text{if } x>3 \\ 6 & \text{if } x=3 \\ x^2-6 & \text{if } x<3\end{cases}$

(32) $q(x)=\begin{cases}|x-1| & \text{if } x\leqslant-1 \\ 2x^2 & \text{if } x>-1\end{cases}$

(33) $g(x)=\dfrac{x^2-3x-4}{x^2-1}$

(34) $f(x)=\begin{cases}\dfrac{2}{x-1} & \text{if } x<2 \\ \sqrt{6-x} & \text{if } x\geqslant2\end{cases}$

$$(35)\ g(x) = \begin{cases} \dfrac{3}{x^2 - 4} & \text{if} \quad x > 1 \\ x^2 - 1 & \text{if} \quad x \leqslant 1 \end{cases}$$

$$(36)\ f(x) = \begin{cases} \dfrac{2x^2 + 11x + 14}{x^2 + 7x + 10} & \text{if} \quad x < -2 \\ \sqrt{x + 3} & \text{if} \quad -2 \leqslant x \leqslant 6 \\ \dfrac{3x^2 - 13x - 30}{x^2 - 10x + 24} & \text{if} \quad x > 6 \end{cases}$$

For the following questions, find all the possible values of k such that the given function is continuous over $\mathbb{R}$.

$$(37)\ f(x) = \begin{cases} 12 & \text{if} \quad x \leqslant -3 \\ kx + 3 & \text{if} \quad -3 < x < 5 \\ -12 & \text{if} \quad x \geqslant 5 \end{cases}$$

$$(38)\ g(x) = \begin{cases} 3x - 4k & \text{if} \quad x \geqslant 5 \\ 2x + 9 & \text{if} \quad x < 5 \end{cases}$$

$$(39)\ h(x) = \begin{cases} -x^2 - 5k & \text{if} \quad x < 2 \\ k^2 - \dfrac{20}{x} & \text{if} \quad x \geqslant 2 \end{cases}$$

$$(40)\ f(x) = \begin{cases} \dfrac{x^2 + 2x - 3}{x - 1} & \text{if} \quad x \neq 1 \\ k^2 & \text{if} \quad x = 1 \end{cases}$$

$$(41)\ F(x) = \begin{cases} kx^2 + 2k^2 x - 4 & \text{if} \quad x \leqslant 1 \\ 4kx^2 + k^2 x + 6 & \text{if} \quad x > 1 \end{cases}$$

$$(42)\ G(x) = \begin{cases} x^2 + k^2 x & \text{if} \quad x \leqslant 1 \\ 5k + 7x & \text{if} \quad x > 1 \end{cases}$$

ANSWERS:

(1) ∞

(2) $-\infty$

(3) $-\infty$

(4) $-\infty$

(5) $-\infty$

(6) $-\infty$

(7) $-\infty$

(8) $-\infty$

(9) $7/5$

(10) $2/3$

(11) $-\infty$

(12) 5

(13) $-1/2$

(14) $7/4$

(15) $x = -1,\ x = 1$

(16) $x = 1$

(17) $x = 1$

(18) $x = 1,\ x = -2$

(19) $x = -6,\ x = 0$

(20) $x = -3,\ x = 3$

(21) $x = -2$

(22) $x = -1$

(23) $x = 0,\ x = 3$

(24) $x = 4,\ x = 0$

(25) $x = 3,\ x = 4$

(26) $x = 2$

(27) $x = 1$

(28) No discontinuity.

(29) $x = -4,\ x = 2$

(30) $x = -2$

(31) $x = 3$

(32) No discontinuity.

(33) $x = 1,\ x = -1$

(34) $x = 1$, discontinuous if $x > 6$

(35) $x = 1,\ x = 2$

(36) $x = -5,\ x = 6$

(37) $k = -3$

(38) $k = -1$

(39) $k = -6$ or $k = 1$

(40) $k = \mp 2$

(41) $k = -2$ or $k = 5$

(42) $k = -1$ or $k = 6$

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