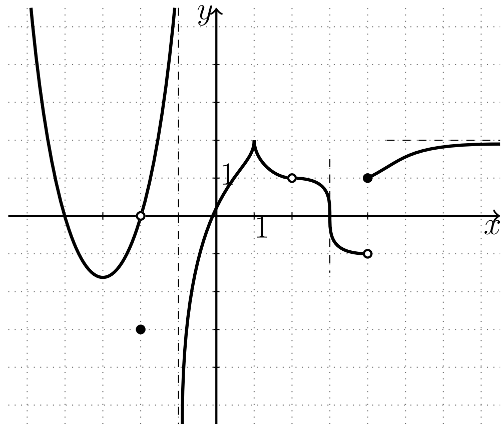
1. (7 points) Consider the function $f(x)$ given by the following graph and evaluate the following expressions. If a limit does not exist, write $\infty, -\infty$ or DNE as appropriate.



(a) $\lim_{x\to -2} f(x) =$

(b) $f(-2) =$

(c) $\lim_{x\to 2} f(x) =$

(d) $\lim_{x\to 4^-} f(x) =$

(e) $\lim_{x\to 4} f(x) =$

(f) $\lim_{x\to -\infty} f(x) =$

(g) $\lim_{x\to \infty} f(x) =$

(h) $\lim_{x\to -4^-} \dfrac{1}{f(x)} =$

(i) $\lim_{x\to -1^+} f(x) =$

(j) $f'(-3) =$

(k) Find the values of x for which f is discontinuous.

(l) Find the values of x for which f is continuous, but not differentiable.

2. (18 points) Evaluate the following limits.

(a) $\lim_{x\to -4} \dfrac{2x^2+5x-12}{x^2+7x+12}$

(b) $\lim_{x\to 2} \dfrac{\dfrac{x+2}{x} - \dfrac{x}{x-1}}{2-x}$

(c) $\lim_{x\to -3^+} \dfrac{x+3}{x^2+6x+9}$

(d) $\lim_{x\to \infty} \dfrac{(2x-1)(3x-2)}{(3x+1)^2}$

(e) $\lim_{x\to -1} \dfrac{x+\sqrt{2x+3}}{2x^2-2}$

(f) $\lim_{x\to 5^-} \dfrac{x^2-6x+5}{x\,|x-5|}$

3. (5 points) Given

$$f(x) = \begin{cases} \dfrac{1}{x+4} & \text{if} \quad x \le 1 \\[2ex] \dfrac{x^2-1}{x^2+8x-9} & \text{if} \quad 1 < x \le 2 \\[2ex] \dfrac{3}{x^2+4} & \text{if} \quad 2 < x, \end{cases}$$

find the value(s) of x where the function is not continuous and justify your answers.

4. (3 points) Find the value(s) of the constant k for which the function f is continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} 2k^2 + 3x & \text{if} \quad x \leq 2 \\ x(9-k) & \text{if} \quad x > 2. \end{cases}$$

5. (5 points) Let $f(x) = \sqrt{5-3x}$.

(a) State the limit definition of the derivative.

(b) By using the limit definition of the derivative, find $f'(x)$.

(c) By using the rules of differentiation, confirm that your previous answer is correct.

6. (15 points) Find the derivatives of the following functions. Do not simplify your answers.

(a) $y = 3^x + \sqrt[5]{x^3} + \csc x - 5\log_3 x + e^2$

(b) $y = \dfrac{2^{5x-7}}{3x + \cot(2x)}$

(c) $y = \sqrt{\sec(x^2 - \ln x)}$

(d) $y = 6(xe^x + 1)^3 + x^2$

(e) $y = (\ln(x) + e^{3x})^{\sin(x)}$

7. (3 points) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x+1)^4 \ln(x)}{e^{3x}\cos^2(5x)}$.

8. (3 points) Find the values of x at which the tangent line to the graph of $f(x) = (2x+4)^3(3x-1)^4$ is horizontal.

9. (4 points) Write the equation of the tangent line to the curve $xy = (x+y)^3 - x$ at the point $(-1, 0)$.

10. (3 points) Find the 117$^{\text{th}}$ derivative of $f(x) = 5x^{17} + e^{(3-5x)} + \sin(x+1)$.

11. (4 points) Find the absolute extrema of $f(x) = e^{2x}(x^2 - 2)$ on the interval $[-1, 3]$.

12. (4 points) Use the second derivative test to find the relative extrema of $f(x) = \dfrac{4x^3 + 125}{x}$.
If the test is inconclusive, state this clearly.

13. (10 points) Consider $f(x) = \dfrac{(5x-14)(x+2)}{(x-2)^2}$, with $f'(x) = -\dfrac{16(x-4)}{(x-2)^3}$, and $f''(x) = \dfrac{32(x-5)}{(x-2)^4}$.
Determine the following geometric features of the graph of $f(x)$.

(a) the domain of f,

(b) all vertical and horizontal asymptotes,

(c) all $x-$ and $y-$intercepts,

(d) the intervals on which f is increasing and decreasing,

(e) all local extrema of f,

(f) the intervals on which f is concave up and concave down,

(g) the inflection points of f,

(h) **sketch the graph on the next page**, clearly label any important points on the graph.

14. (2 points) Let the price function for an item be $p(x) = -3x^2 + 600x$.

(a) Find the marginal revenue when production is 300 units.

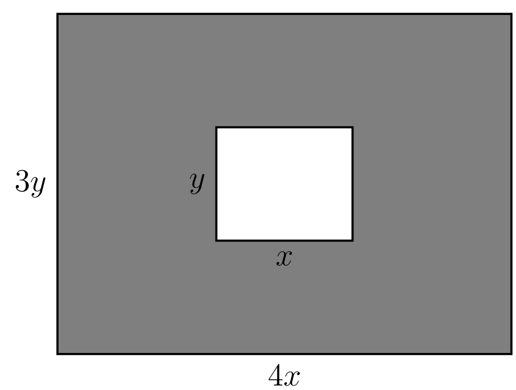
(b) Interpret the result in part (a).

15. (5 points) Urban Freedom is a bike rental shop with many locations throughout the city. Urban Freedom rents a total of 150 bikes per day throughout the city at a rate of \$40 per day. The manager has noticed that for each one dollar decrease in rate, five more bikes are rented.

(a) At what rate should the bikes be rented in order to maximize revenue?

(b) What is the maximum revenue?

16. (5 points) The city of Montreal is creating a new park in Griffins Square, which will consist of a monument commemorating Sir John Abbott, surrounded by a rectangular grass-covered area. The city will enclose the monument, as well as the exterior perimeter of the grass, with fencing. The most aesthetically pleasing proportions for this project are shown in the figure below, where the gray shaded region represents the grass-covered area, and the monument is in the middle. Find the maximum possible area of grass that can be obtained if the city has exactly 800 meters of fencing to use for this project.



17. (4 points) The demand for Falkor's delicious vegan wafers is given by $2x = 1944 - 2p^2$.

(a) Find the price elasticity of demand function (as a function of p).

(b) Find the elasticity when the price is \$10. Is demand elastic, inelastic, or unit elastic? Should the price be raised or lowered to increase revenue?

(c) What price must be charged for maximum revenue?

Answers.

1.

(a) 0 (c) 1 (e) DNE (g) 2 (i) $-\infty$

(b) -3 (d) -1 (f) ∞ (h) ∞ (j) 0

(k) $x=-2, x=-1, x=2, x=4$ (l) $x=1, x=3$

2.

(a) 11 (b) $-\frac{1}{2}$ (c) ∞ (d) $\frac{2}{3}$ (e) $-\frac{1}{2}$ (f) $-\frac{4}{5}$

3. $x=1$ no discontinuity, $x=-4$ infinite discontinuity, $x=2$ jump discontinuity

4. $k=-3, k=2$

5. $f'(x) = -\dfrac{3}{2\sqrt{5-3x}}$

6.

(a) $y' = 3^x \ln 3 + \dfrac{3}{5}x^{-\frac{2}{5}} - \csc x \cot x - \dfrac{5}{\ln 3}\dfrac{1}{x}$

(b) $y' = \dfrac{2^{5x-7}\ln 2 \cdot 5(3x+\cot(2x)) - 2^{5x-7}(3-\csc^2(2x)\cdot 2)}{(3x+\cot(2x))^2}$

(c) $y' = \dfrac{1}{2}\left(\sec\left(x^2-\ln x\right)\right)^{-\frac{1}{2}} \sec(x^2-\ln x)\tan(x^2-\ln x)\left(2x-\dfrac{1}{x}\right)$

(d) $y' = 18(xe^x+1)^2(e^x+xe^x)+2x$

(e) $y' = \left(\cos(x)\ln(\ln(x)+e^{3x}) + \dfrac{\sin(x)}{\ln(x)+e^{3x}}\left(\dfrac{1}{x}+3e^{3x}\right)\right)\left(\ln(x)+e^{3x}\right)^{\sin(x)}$

7. $y' = \left(\dfrac{4}{x+1} + \dfrac{1}{x\ln(x)} - 3 + \dfrac{10\sin(5x)}{\cos(5x)}\right)\dfrac{(x+1)^4\ln(x)}{e^{3x}\cos^2(5x)}$

8. $f'(x) = 6(2x+4)^2(3x-1)^3(7x+7)$, $x=-2, x=\dfrac{1}{3}, x=-1$

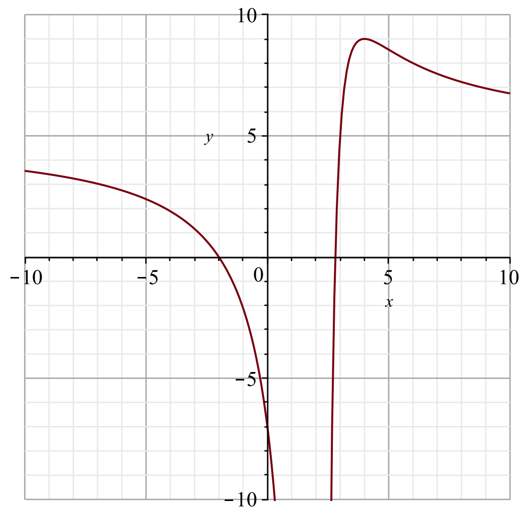
9. derivative of y: $y' = \dfrac{3(x+y)^2-y-1}{x-3(x+y)^2}$, equation of tangent line: $y = -\dfrac{1}{2}x - \dfrac{1}{2}$

10. $\dfrac{d^{117}f}{dx^{117}} = e^{(3-5x)}\cdot(-5)^{117} + \cos(x+1)$

11. Abs. max: $f(3) = 7e^6$ at $x=3$. Abs. min: $f(1) = -e^2$ at $x=1$

12. Local min at $x=\dfrac{5}{2}$, $f\left(\dfrac{5}{2}\right) = 75$. No local max.

13.



(a) Domain: $(-\infty, 2) \cup (2, \infty)$

(b) vertical asymptote: $x = 2$, horizontal asymptote: $y = 5$

(c) y-int: $y = \overset{-7}{\cancel{7}}$, x-int: $x = \frac{14}{5}, x = -2$

(d) increasing: $(2, 4)$, decreasing: $(-\infty, 2) \cup (4, \infty)$

(e) local max $x = 4$, $f(4) = 9$, local max: none

(f) concave up: $(5, \infty)$, concave down: $(-\infty, 2) \cup (2, 5)$

(g) point of inflection: $(5, \frac{77}{9})$

14. $R'(300) = -450000$. If the production level is increased by one unit at $x = 300$, the revenue will drop approximately by \$450000.
15. (a) \$35 (b) \$6125
16. Max area: $22000m^2$ with dimensions $x = 40m$, $y = 50m$
17.

(a) $E(p) = \dfrac{2p^2}{972 - p^2}$

(b) $E(10) = \dfrac{200}{872} < 1$ the demand is inelastic, the price should be raised to increase revenue.

(c) $p = \$18$