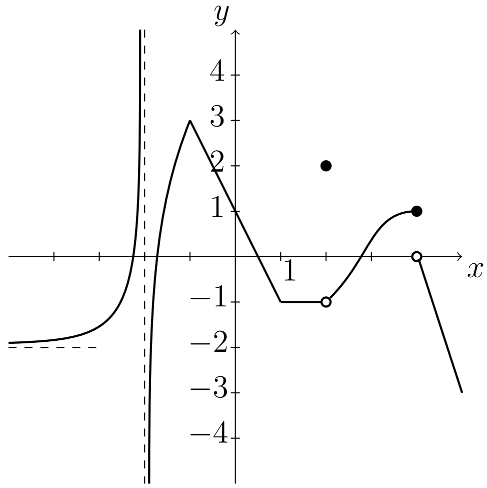
1. (4 points) Given the graph of f below, answer each of the following questions. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.



(a) $\lim\limits_{x\to 2} f(x)$

(b) $\lim\limits_{x\to 4^+} f(x)$

(c) $f(4)$

(d) $f'(1.5)$

(e) $\lim\limits_{x\to \infty} f(x)$

(f) $\lim\limits_{x\to -2^+} f(x)$

(g) $\lim\limits_{x\to -2} f(x)$

(h) List all x-values where the function is not differentiable. Justify your answer.

2. (9 points) Evaluate the following limits. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\lim\limits_{x\to -3} \dfrac{x^2-4}{x(x+3)^2}$

(b) $\lim\limits_{x\to 5} \dfrac{1-\sqrt{6-x}}{x-5}$

(c) $\lim\limits_{x\to 4^-} \dfrac{|2x-8|}{x^3-4x^2}$

3. (5 points) Use the definition of continuity to determine the point(s) of discontinuity for the following function.

$$f(x) = \begin{cases} \dfrac{4}{x-1} & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ \dfrac{x+26}{x+5} & \text{if } x > 2 \end{cases}$$

4. (9 points) Calculate y' given each of the following equations. **Do not simplify your answers.**

(a) $y = e^{3x^2}\left[\dfrac{3x}{2} - \sqrt{x^3} + \log_3\left(\dfrac{\pi}{4}\right)\right]$

 Question 4 continues on the next page.

(b) $y = (4\sin(x^3) + 6^x)^4$

(c) $y = \dfrac{5e^{4x+7} + 9x}{\cos(x^4) - 7x^3}$

5. (3 points) Compute the slope of the tangent line to the curve $4x^3y^5 = e^y - (x+y)^2$ at the point $(1, 0)$.

6. (3 points) Compute the second derivative of $f(x) = \dfrac{\sqrt{x}\tan(3x) + x^2 - \pi}{\sqrt{x}}$.

7. (10 points) Answer parts (a) - (h) using the information about f given below.

$$f(x) = \frac{(2x-8)(x+2)}{x^2} \qquad f'(x) = \frac{4(x+8)}{x^3} \qquad f''(x) = \frac{-8(x+12)}{x^4}$$

(a) State the domain of f.

(b) Find the x-intercept(s) and y-intercept of f, if any exist.

(c) Find all vertical and horizontal asymptotes of f, if any exist.

(d) State the interval(s) where f is increasing and where f is decreasing.

(e) Find all local/relative extrema of f, if any exist.

(f) State the interval(s) where f is concave up and where f is concave down.

(g) Find all inflection points of f, if any exist.

(h) Use your answers from the preceding parts to sketch the graph $y = f(x)$ on the grid provided.

8. (4 points) The **total cost** in dollars to manufacture x units of a product is given by

$$C(x) = \frac{2}{3}x^3 + 20x + 36000$$

How many units should be manufactured in order to minimize the **average cost** per unit?

9. (4 points) Find the absolute extrema for the function $f(x) = x - \sqrt{x}$ on the interval $[0, 1]$.

10. (5 points) A student trip costs \$800 per ticket if exactly 30 students take part. However, if more than 30 students take part, then the ticket price is reduced by \$20 for each additional student. Note that the maximum number of students is 60 and if fewer than 30 students take part, then the trip will be cancelled.

(a) How many students should take part in order to maximize the revenue collected?

(b) What would the ticket price be in this case?

11. (4 points) Samsung manufactures a series of 55 inch flat screen LCD televisions. The quantity x of these televisions demanded each week is related to the wholesale unit price p by the equation

$$x = 45 - \frac{p^2}{5}, \quad (0 \le p \le 15)$$

where x is the quantity demanded per week (measured in units of hundred) and p is the unit price (measured in dollars).

(a) Compute the price elasticity of demand function.

(b) Is the demand elastic or inelastic when $p = 10$?

(c) If the unit price is lowered slightly from \$10, will the total revenue increase or decrease?

 Question 11 continues on the next page.

ANSWERS:

1. (a) -1 (b) 0 (c) 1 (d) 0 (e) ∞ (f) $-\infty$ (g) DNE (h) Discontinuities $x = -2, 2, 4$; Corners $x = -1, 1$

2. (a) $-\infty$ (b) $\frac{1}{2}$ (c) $-\frac{1}{8}$

3. Removable discontinuity at $x = 2$ and infinite discontinuity at $x = 1$

4. (a) $y' = 6xe^{3x^2}\left(\frac{3x}{2} - x^{3/2} + \log_3\left(\frac{\pi}{4}\right)\right) + e^{3x^2}\left(\frac{3}{2} - \frac{3}{2}x^{1/2}\right)$

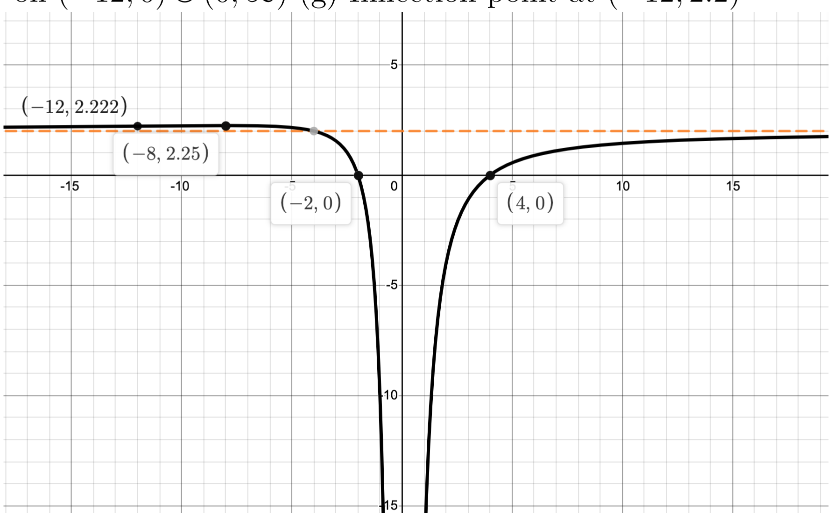
(b) $y' = 4(4\sin(x^3) + 6^x)^3(12x^2\cos(x^3) + (\ln 6)6^x)$

(c) $y' = \dfrac{(20e^{4x+7} + 9)(\cos(x^4) - 7x^3) - (5e^{4x+7} + 9x)(-4x^3\sin(x^4) - 21x^2)}{(\cos(x^4) - 7x^3)^2}$

5. Slope is -2

6. $f''(x) = 18\sec^2(3x)\tan(3x) + \frac{3}{4}x^{-1/2} - \frac{3\pi}{4}x^{-5/2}$

7. (a) All $x \neq 0$ (b) No y-intercept; x-intercepts at $(4, 0)$ and $(-2, 0)$ (c) VA at $x = 0$; HA at $y = 2$ (d) Increasing on $(-\infty, -8) \cup (0, \infty)$; decreasing on $(-8, 0)$ (e) Local max at $(-8, 2.25)$ (f) CU on $(-\infty, -12)$; CD on $(-12, 0) \cup (0, \infty)$ (g) Inflection point at $(-12, 2.\overline{2})$



8. 30 units

9. Absolute max of 0 occurs at $x = 0$ and $x = 1$; Absolute min of $-\frac{1}{4}$ occurs at $x = \frac{1}{4}$

10. (a) 35 students (b) \$700 ticket price

11. (a) $E(p) = \dfrac{2p^2}{225 - p^2}$ (b) $E(10) = 1.6 > 1$ so demand is elastic (c) Total revenue will increase