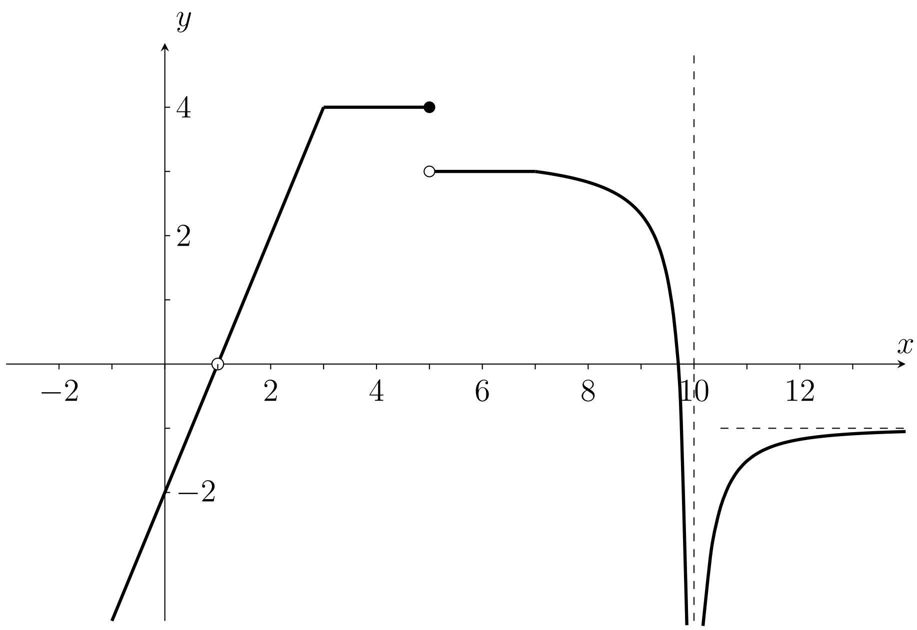
1. Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.



(a) $\lim_{x\to 1} f(x) =$

(b) $\lim_{x\to 5^-} f(x) =$

(c) $\lim_{x\to 5} f(x) =$

(d) $\lim_{x\to 10} f(x) =$

(e) $\lim_{x\to \infty} f(x) =$

(f) $f'(0) =$

(g) State the value(s) of x where f is discontinuous.

(h) State the value(s) of x where f is continuous but not differentiable.

2. Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.

(a) $\lim_{x\to 3} \dfrac{x^2+2x-15}{2x^2-x-15}$

(b) $\lim_{x\to -2} \dfrac{x^2+2x}{3-\sqrt{7-x}}$

(c) $\displaystyle\lim_{x\to -2^-} \frac{|x+2|}{3x^2-12}$

(d) $\displaystyle\lim_{x\to 6} \frac{\dfrac{2}{x+4}-\dfrac{3}{2x+3}}{6-x}$

(e) $\displaystyle\lim_{x\to -\infty} \frac{3x^2-x-4x^3}{x^2(x^2+1)}$

3. Define

$$f(x)=\begin{cases}\frac{-2x-1}{x+2} & x\leqslant -1\\ \sqrt{x+1} & -1<x\leqslant 3\\ 2x-4 & x>3\end{cases}$$

find all x-values where f is not continuous and justify your answer.

4. Define

$$f(x)=\begin{cases}a^2x^2-9 & x<-1\\ \frac{2ax+6}{2x+3} & x\geqslant -1\end{cases}$$

Find the value(s) for a such that f is continuous everywhere.

5. Find the derivative of each of the following functions. Do not simplify your answers.

(a) $f(x)=\frac{-2}{x^3}+2\sqrt[3]{x^4}+\pi^x+e^{\pi^2}$

(b) $y=\sec(4x^3+5)+x\sin^3(x)$

(c) $y=\sqrt{\log_2(3x^2+1)+3x^2}$

(d) $y=\dfrac{4^{2x-1}}{\cot(x)-3e^x}$

(e) $y=\big(\cos(x)\big)^{x^2}$

6. Given $f(x)=3x-x^2$,

(a) Use the **<u>limit definition of the derivative</u>** to find the derivative of $f'(x)$.

(b) Find an equation of the tangent line to the curve $y=f(x)$ at $x=2$.

7. Given $(x+4y)^3=1-2xy$, find $y'=\dfrac{dy}{dx}$.

8. Given $f(x)=3\sqrt[3]{x}(x-12)$, find all critical numbers of f.

9. Given $f(x)=2x^3-3x^2-12x$, find the absolute extrema of the function f on $[0,5]$.

10. (12 points) Given

$$f(x)=\frac{(x+4)(x-2)}{x^2}\qquad f'(x)=\frac{-2(x-8)}{x^3}\qquad f''(x)=\frac{4(x-12)}{x^4}$$

(a) Find the domain of f,

(b) Find the x- and y-intercepts of f,

(c) Find any vertical and horizontal asymptotes of f,

(d) Find the intervals of increase and decrease of f,

(e) Find any local extrema of f,

(f) Find the intervals of concavity of f,

(g) Find any points of inflection of f,

(h) Use your answers from the previous parts to sketch a graph of f on the grid below. Choose the scale of your axes carefully. Show all relevant information on the graph.

11. All units in a 30-unit apartment building are rented out when the monthly rent is set at \$1000/month. A survey reveals that one unit becomes vacant with each \$50 increase in rent. Which rent maximizes the monthly revenue?

12. The demand function for phone cases is given by $x = 400 - 2p^2$.

(a) Find the price elasticity of demand function.

(b) When $p = 10$ is demand elastic, inelastic or unitary?

(c) At the price of \$10, if the price is increased by 10%, how would the demand be affected?

Solutions:

1. (a) 0 (b) 4 (c) DNE (d) $-\infty$ (e) -1 (f) 2 (g) $x = 1, 5, 10$ (h) $x = 3$

2. (a) $\frac{8}{11}$ (b) -12 (c) $\frac{1}{12}$ (d) $-\frac{1}{150}$ (e) 0

3. f is not continuous at $x = -2, -1$.

4. $a = -5, 3$

5. (a) $f'(x) = 6x^{-4} + \frac{8}{3}x^{\frac{1}{3}} + \pi^x \ln \pi$

   (b) $y' = 12x^2 \sec(4x^3+5)\tan(4x^3+5) + \sin^3(x) + 3x\sin^2(x)\cos(x)$

   (c) $y' = \frac{1}{2}\left[\log_2(3x^2+1) + 3x^2\right]^{-\frac{1}{2}}\left[\frac{6x}{(3x^2+1)\ln 2} + 6x\right]$

   (d) $y' = \frac{4^{2x-1}\ln(4)2(\cot(x)-3e^x) - 4^{2x-1}(-\csc^2(x)-3e^x)}{[\cot(x)-3e^x]^2}$

   (e) $y' = \left[\cos(x)\right]^{x^2}\left[2x\ln(\cos(x)) - \frac{x^2\sin(x)}{\cos(x)}\right]$

6. (a) $f'(x) = 3 - 2x$ (b) $y = -x + 4$

7. $y' = \frac{-2y-3(x+4y)^2}{12(x+4y)^2+2x}$

8. $x = 0$ and $x = 3$

9. Abs. max $(5, 115)$ Abs. min $(2, -20)$

10. (a) $(-\infty, 0) \cup (0, \infty)$

    (b) x-intercepts: $(-4, 0), (2, 0)$. No y-intercepts.

    (c) Vertical asymptote: $x = 0$; Horizontal asymptote: $y = 1$.

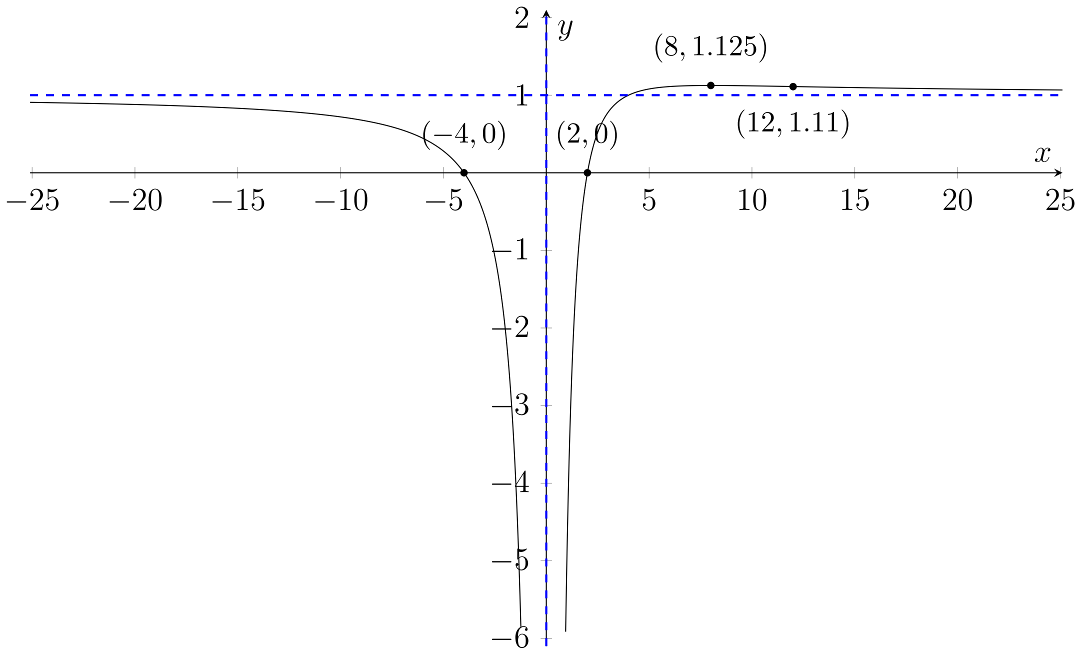
    (d) f is increasing on $(0, 8)$ and decreasing on $(-\infty, 0) \cup (8, \infty)$

(e) f has a local max at $(8, 1.125)$

(f) f is concave up on $(12, \infty)$ and concave down on $(-\infty, 0) \cup (0, 12)$

(g) f has a point of inflection $(12, 1.11)$

(h)



11. The rent that would maximize the monthly revenue is \$1250

12. (a) $E(p) = \frac{4p^2}{400-2p^2}$

 (b) The demand is elastic since $E(10) = 2 > 1$.

 (c) The demand would decrease by 20%.