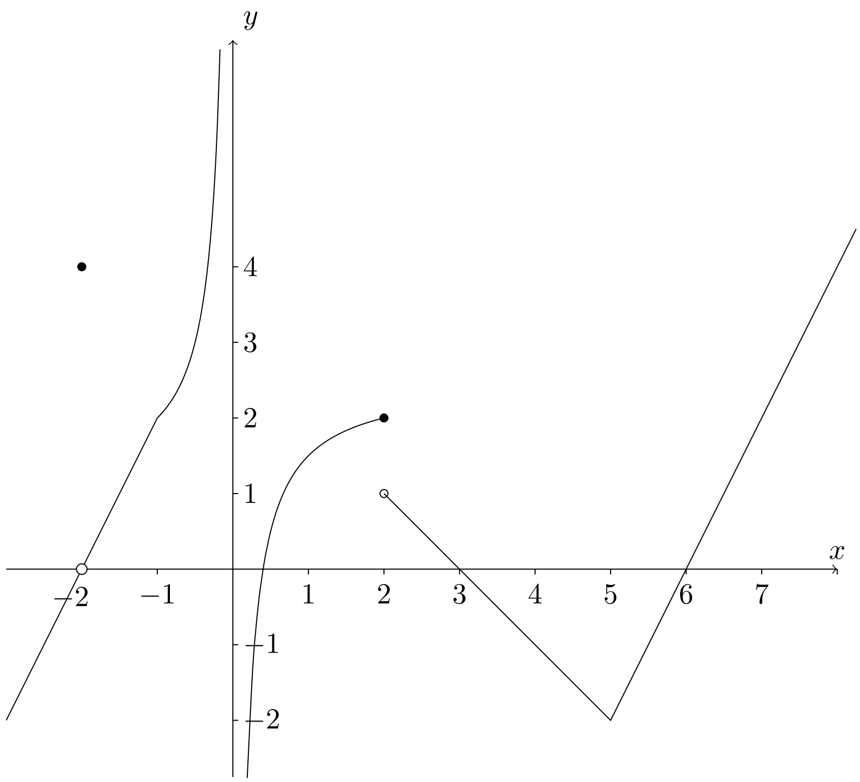
(Marks)

(5) 1. Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



a) $\lim_{x\to 2^-} f(x) =$ b) $\lim_{x\to 2^+} f(x) =$ c) $f(2) =$

d) $\lim_{x\to -2} f(x) =$ e) $\lim_{x\to 0^-} f(x) =$ f) $f(3) =$

g) $\lim_{x\to 0^+} f(x) =$ h) $\lim_{x\to 5} f(x) =$ i) $\lim_{x\to \infty} f(x) =$

j) Is $f(x)$ continuous at $x = -2$?

2. Evaluate the following limits. Identify the limits that do not exist, and use $\pm\infty$ as appropriate.

(3) a) $\displaystyle\lim_{x\to -3} \frac{x^2-9}{x^2+4x+3}$

(3) b) $\displaystyle\lim_{x\to 2} \frac{x-2}{\sqrt{x+2}-2}$

(3) c) $\displaystyle\lim_{x\to 2} \frac{\frac{1}{x+2}-\frac{1}{4}}{x-2}$

(3) d) $\displaystyle\lim_{x\to -3} \frac{-x(x-3)}{x^2-9}$

(2) e) $\displaystyle\lim_{x\to -\infty} \frac{(2x-3)^2}{x^2-1}$

(Marks)

(3) f) $\lim\limits_{x \to -2^-} \dfrac{|x+2|}{x^2+2x}$

(3) 3. Find the point(s) of discontinuity for the following function. Justify using the definition of continuity.

$$f(x) = \begin{cases} 2x-1 & \text{if } x \leq 3 \\ \dfrac{5x-1}{x^2-6x+8} & \text{if } x > 3 \end{cases}$$

(3) 4. Find the value(s) for k such that the following function is continuous for all real numbers.

$$f(x) = \begin{cases} \dfrac{x^2-64}{x-8} & \text{if } x \neq 8 \\ k & \text{if } x = 8 \end{cases}$$

(6) 5. Let $f(x) = 3x - 2x^2$.
(a) State the limit definition of the derivative
(b) Use the definition to find the derivative of $f(x)$.
(c) Find the coordinates of the point(s) of $f(x)$ where the tangent line has slope 5.

6. Find the derivative for each of the following functions. Do not simplify.

(3) a) $y = 3^{2x+1} + \log_7(x^2-7) + \sqrt[5]{x^3} - e^3$

(3) b) $y = \left(4x^{-5} - 6x^{-3}\right)\left(\dfrac{x^4}{3} - x\right)$

(3) c) $y = \dfrac{\tan(x)+1}{\sec(2x)}$

(3) d) $y = \left(\dfrac{x^5-7x^2}{3x^4+9}\right)^6$

(4) e) $y = (x^2+5x)^{x^3-7x}$

(4) f) $y = \ln\left(\dfrac{\cos^3(x)(3x^2-8)^7}{\sqrt{x}(2x-1)^{11}}\right)$

(4) g) $y = \ln\left(\sin\left(e^{\sqrt{x}}\right)\right)$

(Marks)

(6) 7. Let $x^2y^3 + 3xy^5 - 4x + 3y^2 = 7 + 7x^2 - 4y$.

a) Find $\dfrac{dy}{dx}$

b) Find the equation of the tangent line to the curve at the point $(0, 1)$

(10) 8. Given $f(x) = \dfrac{x^2 + 2x}{(x+4)(x-2)}$ with $f'(x) = \dfrac{-16(x+1)}{(x+4)^2(x-2)^2}$ and $f''(x) = \dfrac{48(x^2+2x+4)}{(x+4)^3(x-2)^3}$,

(a)Find the x- and y- intercepts (if any).

(b)Find the vertical and horizontal asymptotes (if any).

(c)Give the intervals where $f(x)$ is increasing and decreasing, and the relative extrema (if any).

(d)Give the intervals where $f(x)$ is concave up and concave down, and the points of inflection (if any).

(e)Sketch a labelled graph of $f(x)$ on the next page.

(3) 9. Use the second derivative test to find all the relative (local) extrema of

$$f(x) = x^4 + \frac{8}{3}x^3 - 30x^2 + 7$$

(3) 10. Find the absolute (global) extrema of $f(x) = (x^2 - 4x - 5)^3$ on the interval $[-2, 4]$.

(5) 11. A rectangular pig pen to be built and divided into 3 equal parts using 1000 feet of fencing. What dimensions will maximize the total area of the pig pen?

(5) 12. Given the demand function $p = -0.009x + 30$ and the cost function $C(x) = 0.001x^2 + 100x + 4000$,
a) Determine the average cost function.
b) Determine the production level that will result in the minimum average cost.
c) Determine the unit price that will result in the minimum average cost.

(5) 13. A travel company charges \$600 per person if exactly 20 people sign up for their five-day bus tour to the Maritimes. However, if more than 20 people sign up, then each fare is reduced by \$4 for each additional passenger (up to a maximum capacity of 90 people). If fewer than 20 people sign up for the tour, then it will be cancelled.

a) How many passengers will results in the maximum revenue for the travel company?
b) What is the maximum revenue?
c) What would the fare per passenger be in this case?

(Marks)

(5) 14. The demand function of a product is given by $p = 200 - 5\sqrt{x}, 0 \leq x \leq 1600$, where x is the number of units produced per day and p is the unit price. Currently 1024 units are produced every day.

a) Determine the price elasticity of demand function $\eta(x)$.
b) Determine the price elasticity of demand at a production level of 1024. Is demand elastic?
c) At a production level of 1024 units, how will the current demand will be affected if the unit price is increased by 5%?
d) Determine the production level that would maximize the revenue.

Answers

1.a) 2 b) 1 c) 2 d) 0 e) ∞ f) 0 g) $-\infty$ h) -2 i) ∞ j) No.

2.a) 3 b) 4 c) $\frac{-1}{16}$ d) DNE e) 4 f) $\frac{1}{2}$

3) $x = 3$ (Jump) and $x = 4$ (Vertical Asymptote) 4) $k = 16$

5.a) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ b) $f'(x) = 3 - 4x$ c) $(-\dfrac{1}{2}, -2)$

6.a) $3^{2x+1}(\ln 3)(2) + \dfrac{2x}{(x^2-7)\ln 7} + \dfrac{3}{5}x^{\frac{-2}{5}}$ b) $(-20x^{-6} + 18x^{-4})\left(\frac{x^4}{3} - x\right) + (4x^{-5} - 6x^{-3})(\frac{4x^3}{3} - 1)$

c) $\dfrac{\sec^2 x(\sec(2x)) - (\tan x + 1)(\sec(2x)\tan(2x)(2))}{\sec^2(2x)}$

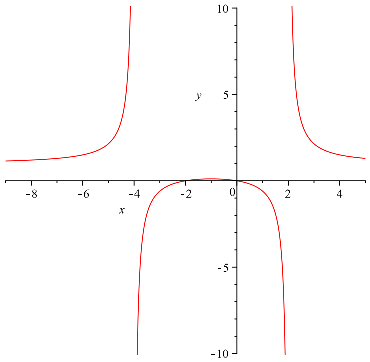
d) $6\left(\dfrac{x^5 - 7x^2}{3x^4 + 9}\right)^5 \left(\dfrac{5x^4 - 14x)(3x^4 + 9) - (x^5 - 7x^2)(12x^3)}{(3x^4 + 9)^2}\right)$

e) $\left[(3x^2 - 7)\ln(x^2 + 5x) + (x^3 - 7x)\left(\dfrac{2x + 5}{x^2 + 5x}\right)\right](x^2 + 5x)^{x^3 - 7x}$

f) $-3\tan x + 7\left(\dfrac{6x}{3x^2 - 8}\right) - \dfrac{1}{2x} - \dfrac{22}{2x - 1}$ g) $y = \dfrac{1}{(\sin(e^{\sqrt{x}}))}(\cos(e^{\sqrt{x}})(e^{\sqrt{x}})(\dfrac{1}{2}x^{-1/2})$

7.a) $y' = \dfrac{14x + 4 - 3y^5 - 2xy^3}{3x^2y^2 + 15xy^4 + 6y + 4}$ b) $y = \dfrac{x}{10} + 1$

8.a) Intercepts $(0,0),(-2,0)$ b) VA:$x=-4, x=2$ HA: $y=1$
c) INC:$(-\infty,-4)\cup(-4,-1)$ DEC:$(-1,2)\cup(2,\infty)$ Local Max: $(-1,\frac{1}{9})$, no local Mins
d) CU:$(-\infty,-4)\cup(2,\infty)$ CD:$(-4,2)$, no inflection points.



9) Relative Max: $(0,7)$, Relative Mins: $(3,-110),(-5,\frac{-1354}{3})$
10)Abs Max: $(-2,343)$, Abs Min: $(2,-729)$ 11) Dimensions 250 ft $\times$ 125 ft
12. a) $\bar{C}=0.001x+100+4000x^{-1}$ b) $x=2000$ c) \$12
13. a) 85 passengers b) \$28900 c) \$340

14.a) $\eta=\dfrac{200-5x^{1/2}}{\frac{-5}{2}x^{1/2}}$ b) $\eta(1024)=-0.5$ Inelastic c) It will decrease by 2.5%. d) $x\approx 711$