1. Evaluate the following limits. If a limit is does not exist, use "does not exist", ∞ or $-\infty$ where appropriate.

(a) (2 points) $\lim\limits_{x\to -3^-} \dfrac{x^2+4x}{(x+3)^2}$

(b) (3 points) $\lim\limits_{x\to 2} \dfrac{\frac{1}{3} - \frac{x}{4x-2}}{x-2}$

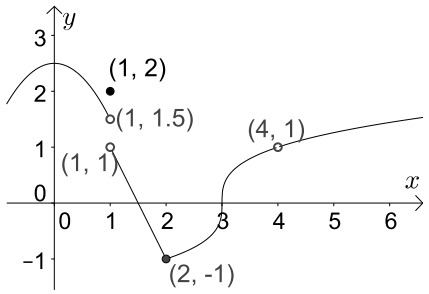
(c) (3 points) $\lim\limits_{x\to 7} \dfrac{3x^2-11x-70}{49-x^2}$

(d) (3 points) $\lim\limits_{x\to 4^-} \dfrac{8+2x-x^2}{|4-x|}$

(e) (2 points) $\lim\limits_{x\to \infty} \dfrac{(x^3+4x)(x^2+9)}{14+6x-x^4}$

2. (4 points) Use the graph of f below to answer the following questions. If a limit is does not exist, use "does not exist", ∞ or $-\infty$ where appropriate.

Note: f has a vertical tangent at the point $(3, 0)$.



(a) Find the interval(s) where f is continuous.

(b) Find the x-value(s) where f is continuous, but not differentiable.

(c) Find $\lim\limits_{x\to 4} f(x)$.

(d) Find $f(1)$.

(e) Find $\lim\limits_{x\to 1^-} f(x)$.

(f) Find $f(4)$.

3. (a) (2 points) Give the definition of continuity of a function f at $x = c$.

(b) (3 points) At which values of x is the function

$$f(x) = \begin{cases} \dfrac{5x+10}{(x+2)(x-5)} & \text{if } x \le 0 \\ 2x^2-3x-1 & \text{if } x > 0 \end{cases}$$

discontinuous? Justify your answers by referencing the definition of continuity.

4. (3 points) Find all values of k such that the following function is continuous everywhere:

$$f(x) = \begin{cases} k^2x^2 + 3kx - 8 & \text{if } x < 2 \\ kx & \text{if } x \geq 2 \end{cases}$$

5. (a) (1 point) State the limit definition of the derivative of a function f.

(b) (3 points) Use the definition from part a) to find the derivative of $f(x) = \sqrt{3 - 2x}$.

6. Differentiate the following functions. Do not simplify your answers.

(a) (3 points) $y = e^{x^2-4x} + x^\pi - \dfrac{3}{x^5} + 4\sqrt[5]{x^7}$

(b) (3 points) $y = \dfrac{\log_2(x^3)}{\sqrt{x} + 8\sin x}$

(c) (3 points) $y = (7 - 5^{3x+1})^3(\sec(x^3) - 7)$

(d) (3 points) $y = 3(\cos(2x))^{x^2+2}$

7. (5 points) Find an equation of the line tangent to the curve $2x^3y^2 + 4x^2 - 5y^3 + e^{xy+1} = 12$ at $(1, -1)$.

8. (4 points) Find $\dfrac{d^8y}{dx^8}$, given $y = \log_3 e^x - \cos x + 8x^5 + e^{3x-3}$.

[Hint: Simplify and do each term separately.]

9. (4 points) Use logarithmic differentiation to find the derivative of the function $y = \dfrac{(9x^2 - 4)^7\sqrt{3x^4 - 7}}{e^x \ln(x^5)}$.

10. (4 points) Given $y = x^5e^{3x}$, find y', and all the x-values at which the function has a horizontal tangent.

11. (4 points) Find all the vertical and horizontal asymptotes of $f(x) = \dfrac{-x^3 + 5x^2 + 3x}{(2x + 7)(x^2 - 4)}$.

12. Let $f(x) = \sqrt[3]{(x^2 - 64)^2}$.

(a) (3 points) Find all critical numbers of f.

(b) (2 points) Find the absolute extrema of f on the interval $[-10, 20]$.

13. (3 points) **Use the second derivative test** to determine the relative extrema of $f(x) = \frac{5}{3}x^3 + 2x^2 - x + 11$.

14. (10 points) Given

$$f(x) = \frac{1 - x}{x^2} \qquad f'(x) = \frac{x - 2}{x^3} \qquad f''(x) = \frac{2(3 - x)}{x^4},$$

determine:

(a) any x- and y-intercepts,

(b) any vertical and horizontal asymptotes,

(c) the intervals on which f is increasing and decreasing,

(d) any relative maxima and minima of f,

(e) the intervals on which f is concave up and concave down,

(f) any inflection points of f.

(g) Neatly sketch a graph of f, including all pertinent information obtained in the previous parts.

Continue question 14 below:

15. (5 points) A manufacturer of men's shirts determines that their production costs will be \$500 overhead plus \$9 for each shirt produced (that is, $C(x) = 500 + 9x$). It has been determined that the price per shirt p should be given by $p(x) = 30 - 0.2\sqrt{x}$, where x is the number of shirts sold.

(a) How many shirts should be produced in order to maximize profit?

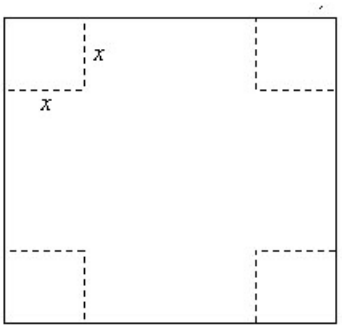
(b) At what price should the shirts be sold in order to maximize profit?

16. (5 points) A hotel has 200 rooms. When the price per room is set to \$92 per night, all rooms are occupied. From past experience, it is known that for each \$2 increase in price, one fewer room will be occupied.

(a) What nightly rate should the hotel charge in order to maximize their revenue?

(b) How many rooms will be occupied when the revenue is maximized?

17. (5 points) An open box is to be made from a 12cm by 12cm piece of cardboard by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made this way.



18. (5 points) Alice starts a business selling photo frames. The demand curve for these photo frames is given by $6p^2 + 2x = 900$.

(a) Determine the price elasticity of demand function η.

(b) What is the price elasticity of demand when the price is \$8? Is the demand elastic, inelastic, or unit elastic?

(c) At the price of \$8, if the price increases by 5%, how will demand be affected?

(d) Determine the price that would maximize the revenue.

Answers:

1. (a) $-\infty$
   (b) $\dfrac{1}{18}$
   (c) $-\dfrac{31}{14}$
   (d) 6
   (e) $-\infty$

2. (a) $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$
   (b) $x = 2$ because there is a cusp and $x = 3$ because there is a vertical tangent.
   (c) 1
   (d) 2
   (e) 1.5
   (f) DNE

3. (a) A function f is continuous at c if $\lim\limits_{x \to c} f(x) = f(c)$.
   (b) f is discontinuous at $x = -2$ because $f(-2)$ does not exist. f is continuous at all other values of x.

4. $k = -2$ or $k = 1$.

5. (a) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
   (b) $f'(x) = \dfrac{-1}{\sqrt{3 - 2x}}$

6. (a) $y' = (2x - 4)e^{x^2 - 4x} + \pi x^{\pi - 1} + \dfrac{15}{x^6} + \dfrac{28}{5} x^{\frac{2}{5}}$
   (b) $y' = \dfrac{(3)(2\sqrt{x})(\sqrt{x} + 8\sin x) - (x \ln 2)(\log_2(x^3))(1 + 16\sqrt{x}\cos x)}{(x \ln 2)(2\sqrt{x})(\sqrt{x} + 8\sin x)^2}$
   (c) $y' = -9\ln(5)(7 - 5^{3x+1})^2(5^{3x+1})(\sec(x^3) - 7) + (7 - 5^{3x+1})^3(\sec(x^3)\tan(x^3)(3x^2))$
   (d) $y' = 3(\cos(2x))^{x^2+2}[2x \ln(\cos(2x)) - 2(x^2 + 2)\tan(2x)]$

7. $y = \dfrac{13}{18}x - \dfrac{31}{18}$

8. $\dfrac{d^8 y}{dx^8} = -\cos x + 3^8 e^{3x-3}$

9. $y' = \left(\dfrac{(9x^2 - 4)^7 \sqrt{3x^4 - 7}}{e^x \ln(x^5)}\right)\left(\dfrac{126x}{9x^2 - 4} + \dfrac{6x^3}{3x^4 - 7} - 1 - \dfrac{1}{x \ln x}\right)$

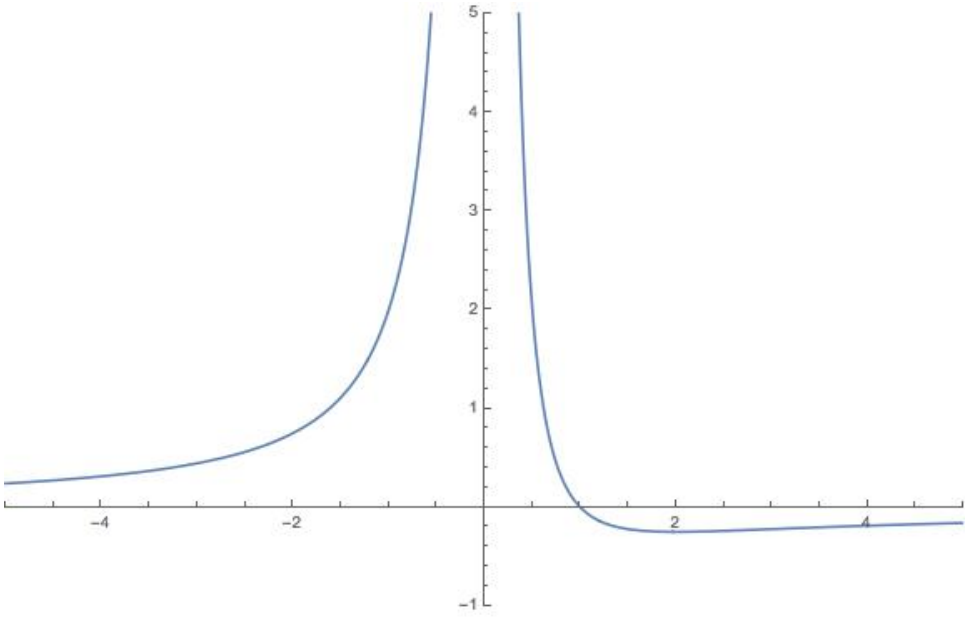
10. $x = -\dfrac{5}{3}, 0.$

11. H.A. at $y = -\dfrac{1}{2}$; V.A. at $x = -\dfrac{7}{2}, -2, 2$.

12. (a) $x = -8, 0, 8$
    (b) Absolute maximum: $(20, \sqrt[3]{(336)^2})$, Absolute minima: $(-8, 0)$ and $(8, 0)$.

13. Local minimum: $x = \dfrac{1}{5}$, Local maximum: $x = -1$.

14. (a) x-intercept: $(1, 0)$; y-intercept: none
    (b) V.A. $x = 0$; H.A. $y = 0$.
    (c) Increasing: $(-\infty, 0) \cup (2, \infty)$; Decreasing: $(0, 2)$.
    (d) Local minimum at $(2, -\frac{1}{4})$.
    (e) Concave up: $(-\infty, 0) \cup (0, 3)$; Concave down: $(3, \infty)$.
    (f) Inflection point: $(3, -\frac{2}{9})$.
    (g) Graph:



15. (a) They should produce 4900 shirts.
    (b) They should sell them at \$16 each.

16. (a) They should charge \$246 per room.
    (b) 123 rooms will be occupied.

17. The volume of the largest box is 128 cm^3.

18. (a) $\eta = 2 - \dfrac{900}{x}$ or $\eta = \dfrac{-6p^2}{450 - 3p^2}$
    (b) $\eta(8) = -\dfrac{64}{43}$; demand is elastic
    (c) Demand will decrease by $\sim 7.44\%$.
    (d) $p = \$7.07$.