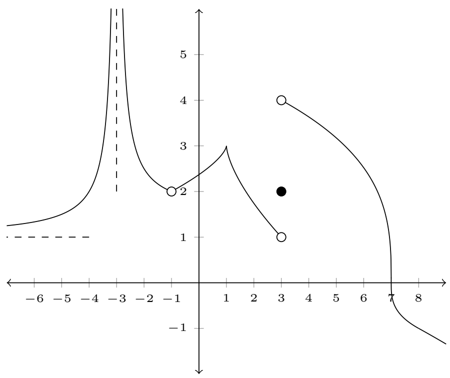
1. The graph of a function f is shown below. Use it to evaluate the following. Use ∞, $-\infty$ or "does not exist" where appropriate. **Note that f has a vertical tangent at $x = 7$.**

   (a) $\lim_{x\to-\infty} f(x)$ (d) $f(-1)$ (g) $\lim_{x\to3^+} \frac{x^2+1}{f(x)}$

   (b) $\lim_{x\to-3} f(x)$ (e) $\lim_{x\to3^-} f(x)$

   (c) $\lim_{x\to-1} f(x)$ (f) $\lim_{x\to3} f(x)$ (h) $\lim_{x\to7^+} \frac{1}{f(x)}$



2. (a) Find the value(s) of x at which the function shown in Question 1 is not continuous. In each case, also identify the type of discontinuity.

   (b) Find the value(s) of x at which the function shown in Question 1 is **continuous, but not differentiable**. In each case, explain why the function fails to be differentiable at that point.

3. Evaluate the following limits.

   (a) $\lim_{x\to2} \frac{2x^2+x-10}{x^2-4}$

   (b) $\lim_{x\to3^-} \left(\frac{2}{|x-3|} - \frac{2}{x-3}\right)$

   (c) $\lim_{x\to1} \frac{\sqrt{2x+1}-\sqrt{4x-1}}{x-1}$

   (d) $\lim_{x\to-2} \frac{x+2}{\frac{3}{x-4}+\frac{1}{x+4}}$

   (e) $\lim_{x\to-\infty} \frac{4x}{\sqrt{9x^2+3}}$

4. Consider the function $f(x) = \dfrac{x^2-5x+6}{x^2-4}$.

   (a) Show that f has a removable discontinuity at $x = 2$.

   (b) Find all vertical and horizontal asymptotes of f.

5. Find the values of a and b that make the following function continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} ae^{-x} + be^{x} & \text{if } x \le 0 \\ ax^2 + b - 1 & \text{if } 0 < x \le 1 \\ \ln x & \text{if } x > 1 \end{cases}$$

6. (a) **Use the limit definition of the derivative** to find $f'(3)$ given that $f(x) = (x-4)^2$.

   (b) Check your answer to part (a) using appropriate differentiation rules.

   (c) Use your answer from the previous parts to find the equation of the line tangent to $f(x)$ and $x = 3$.

7. Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

   (a) $y = \sqrt[6]{x^7} - \dfrac{6}{x^7} + \ln 5 - \sqrt{5}x + e^x - x^e$

   (b) $y = x^2 \sec(\log_5 x)$

   (c) $y = \sqrt[5]{2x^3 - 9\cos(x^2)}$

   (d) $y = \dfrac{4^x}{(3x^2+10x)^3}$

   (e) $xe^y = y \sin x$

   (f) $y = \ln\left[\dfrac{(x^2+1)^4}{x^x \sqrt[3]{\tan x + 4}}\right]$

   (g) $y = x^{\sin x}(3x+2)^7$

8. Calculate the 2003[rd] derivative of $f(x) = e^{-x} + \cos(3x)$.

9. Given the revenue function $R(x) = \dfrac{200x}{x^2+100}$, find the marginal revenue at $x = 20$. Interpret your answer.

10. Find the absolute extrema of $f(x) = \dfrac{x+2}{x^2+12}$ on the interval $[0, 3]$.

11. Find the critical numbers of the function $f(x) = x^{2/3}(x-3)^2$.

12. Find the critical numbers of $f(x) = 3x^4 - 12x^3 + 1$ and apply the second derivative test to each. State your conclusions, or state that the test was inconclusive.

13. You are given the following information about a function f:

    - f is continuous everywhere except at $x = 5$
    - $f(0) = 3$, $f(1) = 1$, $f(3) = -1$
    - f has a vertical asymptote at $x = 5$
    - $\lim_{x\to-\infty} f(x) = 4$, $\lim_{x\to\infty} f(x) = -\infty$
    - $f'(x) > 0$ on $(3, 5)$
    - $f'(x) < 0$ on $(-\infty, 3) \cup (5, \infty)$
    - $f''(x) > 0$ on $(1, 5) \cup (5, \infty)$
    - $f''(x) < 0$ on $(-\infty, 1)$

    Use all of the above information to sketch a graph of f. Clearly label all asymptotes, local extrema and points of inflection.

14. Given
$f(x) = x\sqrt[3]{x+4}$; $f'(x) = \dfrac{4(x+3)}{3\sqrt[3]{(x+4)^2}}$; $f''(x) = \dfrac{4(x+6)}{9\sqrt[3]{(x+4)^5}}$;

    (a) State the intervals of increase and decrease.

    (b) Find the coordinates of each local extremum, and state whether it is a maximum or minimum.

    (c) State the intervals on which the functions is concave up and concave down.

    (d) Find the coordinates of each point of inflection.

15. A company wants to purchase a rectangular advertisement space in a magazine. The magazine requires that the top and bottom margins (2cm each), as well as the side margins (1cm each) of the space be left completely blank. The area of the space offered, including the margins, is 72cm^2. What dimensions should the company request for its advertisement (including the margins) to maximize the printed area?

16. A Bluetooth speaker is currently sold for \$100, with sales averaging 5040 units per month. The marketing department claims that for each \$5 decrease in the price, the demand would increase by 300 speakers per month. (Assume a linear relationship between demand and price.) Find the price that maximizes the revenue.

17. A relation between the production level x and the unit price p in dollars is given by $p = \sqrt{62500 - 2x^2}$.

    (a) Determine the price elasticity of demand function η.

    (b) What is the price elasticity of demand when the production level is 110 units? Is the demand elastic, inelastic, or unit elastic?

    (c) Based on your answer in part (b), how (if at all) should the company adjust the price of the product to increase revenue?

    (d) Determine the price at which demand is unit elastic.

Answers:

1. (a) 1 (d) DNE (g) $5/2$
   (b) ∞ (e) 1
   (c) 2 (f) DNE (h) $-\infty$

2. (a) infinite at $x = -3$; removable at $x = -1$; jump at $x = 3$
   (b) corner at $x = 1$; vertical tangent at $x = 7$

3. (a) $9/4$ (c) $-1/\sqrt{3}$ (e) $-4/3$
   (b) ∞ (d) -3

4. (a) Show that $f(2)$ does not exist, whilst $\lim_{x\to 2} f(x)$ does.
   (b) VA at $x = -2$, HA at $y = 1$

5. $a = -1$, $b = 2$

6. (a) -2 (c) $y - 1 = -2(x - 3)$
   (b) -2

7. (a) $\frac{7}{6}x^{1/6} + 42x^{-8} - \sqrt{5} + e^x - ex^{e-1}$
   (b) $2x\sec(\log_5 x) + \frac{x}{\ln 5}\sec(\log_5 x)\tan(\log_5 x)$
   (c) $\frac{6x^2+18x\sin(x^2)}{5(2x^3-9\cos(x^2))^{4/5}}$
   (d) $\frac{4^x[\ln 4(3x^2+10x)-18x-30]}{(3x^2+10x)^4}$
   (e) $\frac{y\cos x - e^y}{xe^y - \sin x}$
   (f) $\frac{8x}{x^2+1} - \ln x - 1 - \frac{\sec^2 x}{3(\tan x + 4)}$
   (g) $x^{\sin x}(3x+1)^7[\cos(\ln x) + \frac{\sin x}{x} + \frac{21}{3x+2}]$

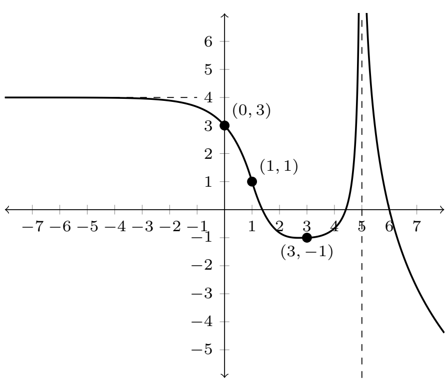
8. $-e^{-x} + 3^{2003}\sin(3x)$

9. $-6/25$

10. Abs. min $(0, \frac{1}{6})$. Abs max $(2, \frac{1}{4})$

11. $x = 0, \frac{3}{4}, 3$

12. inconclusive $x = 0$, local min $x = 3$

13. 

14. (a) Inc: $(-3, \infty)$; Dec: $(-\infty, -3)$
    (b) Local min $(-3, -3)$,
    (c) CU: $(-\infty, -6)$ and $(-4, \infty)$; CD: $(-6, -4)$
    (d) $(-6, 6\sqrt[3]{2})$; $(-4, 0)$

15. 6” × 12”

16. $92

17. (a) $\eta = \frac{x^2-31250}{x^2}$
    (b) ~ -1.58, elastic
    (c) Decrease price.
    (d) $176.78