(Marks)

(5) 1. Use the graph of the function $f(x)$ to find the following. Use ∞, $-\infty$, or DNE where appropriate.

(a) $f(-2) =$ ________________

(b) $f(2) =$ ________________

(c) $\lim\limits_{x\to -2} f(x) =$ ______________

(d) $\lim\limits_{x\to 0} f(x) =$ ______________

(e) $\lim\limits_{x\to 2^+} f(x) =$ ______________

(f) $\lim\limits_{x\to 2^-} f(x) =$ ______________

(g) $\lim\limits_{x\to 2} f(x) =$ ______________

(h) $\lim\limits_{x\to \infty} f(x) =$ ______________

(i) $\lim\limits_{x\to -\infty} f(x) =$ ______________

(j) The intervals on which $f(x)$ is continuous.

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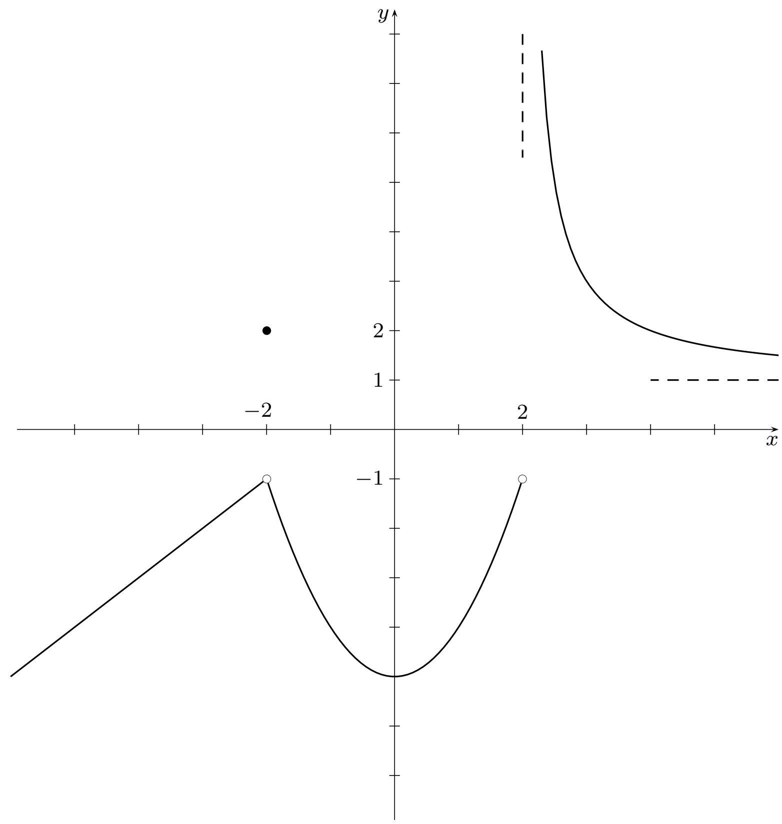

(15) 2. Evaluate the following limits. Use the algebraic methods whenever possible. Identify the limits that do not exist and use ∞ or $-\infty$ where appropriate. Show your work.

(a) $\lim\limits_{x\to 2} \dfrac{\frac{2}{x+2}-\frac{1}{2}}{x-2}$

(b) $\lim\limits_{x\to 2} \dfrac{3x^2-5x-2}{x-2}$

(c) $\lim\limits_{x\to 4^+} \dfrac{x^2-9}{4-x}$

(d) $\lim\limits_{x\to -\infty} \dfrac{8x^3+3x-7}{5x^2+12}$

(e) $\lim\limits_{x\to 4} \dfrac{\sqrt{x+32}-6}{x-4}$

(4) 3. Find the point(s) of discontinuity of the function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{1}{(x+5)(x+1)} & \text{if} \quad x < -2 \\[2ex] \dfrac{x+1}{5} & \text{if} \quad x \geq -2 \end{cases}$$

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(3) 4. Find all the possible value(s) of k so that the function is continuous for all real numbers

$$f(x) = \begin{cases} 3x - 4k & \text{if} \quad x \geq 5 \\ 2x + 9 & \text{if} \quad x < 5 \end{cases}$$

(6) 5. (a) Use the limit definition of the derivative of a function to find $f'(x)$ for $f(x) = \dfrac{3}{2x+5}$.

(b) Check your answer to part a) using the appropriate rule of differentiation.

(18) 6. Find y' for the given functions. **Do not simplify your answer.**

(a) $y = \left(\dfrac{3x-1}{2x+5}\right)^4$

(b) $y = 5^{x^2} + \frac{5}{x^2} + \sqrt{x^5} - \log_5(2x+1) - e^5$

(c) $y = \dfrac{1 + \csc x}{\sin x}$

(d) $y = \sec(\cos(3x))$

(e) $y = e^{\tan x} + e^x \cot(x)$

(f) $y = (x^2+1)^{2x}$

(4) 7. Find $\dfrac{d^2y}{dx^2}$ for the function $y = x^3 e^{x^2}$

(4) 8. Given the implicitly defines function $xy = x^2y - 3x + y$,
find the slope of the tangent line at the point $(0, 0)$.

(4) 9. Use the rules of logarithms to find the derivative of $y = \ln\left(\dfrac{(x^5+3)^{2010}}{e^{3x}\sqrt{6 - x^5 + 3x^2}}\right)$

(4) 10. Find an equation of the tangent line to the curve $y = (x^2+1)\sqrt{2x-3}$ at the point $(2, 5)$.

(4) 11. Use the second derivative test to find the relative extrema of $f(x) = x^3 - 9x^2 + 24x - 20$

(4) 12. Find the absolute extrema of $f(x) = x^4 - 4x^3 + 1$ on the interval $[-1, 4]$.

(10) 13. Given $f(x) = \dfrac{x}{x^2-1}$; $f'(x) = \dfrac{-(x^2+1)}{(x^2-1)^2}$; $f''(x) = \dfrac{2x(x^2+1)}{(x^2-1)^3}$

(a) List the $x-$intercept, $y-$intercept, equations of vertical and horizontal asymptotes (if any), relative extrema, points of inflection, intervals where $f(x)$ is increasing, decreasing, concave up and concave down.

(b) Sketch the graph of $f(x)$.

(Marks)

(5) 14. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of the trees?
(Be sure to use a test to confirm that it is a maximum.)

(5) 15. The demand function for a DVD Player is $p = -0.0005x + 110$ where p is the price per unit and x is the number of units.

(a) Determine the marginal revenue if $x = 80000$. Explain what it means.

(b) If the cost function is given by $C(x) = 5000 + 40x$, determine the profit function.

(c) How many units x should be produced to maximize the profit?

(5) 16. The demand function for a certain product is given by $p = 36 - 2\sqrt{x}$, $0 \leq x \leq 324$ where p is the price per unit and x is the number of units produced.

(a) Find the price elasticity of demand η, if 100 units are produced.

(b) If 100 units are produced, is the demand elastic or inelastic? Justify your answer.

(c) Determine the production level x so that the demand will have unit elasticity.

(Marks)

Answers

1. a) 2 b) undefined c) -1 d) -5 e) $+\infty$ f) -1 g) $D.N.E.$ h) 1

1. i) $-\infty$ j) $x < -2$ or $-2 < x < 2$ or $x > 2$

2. a) $-\frac{1}{8}$ b) 7 c) $-\infty$ d) $-\infty$ e) $\frac{1}{12}$

3. $x = -5$ or $x = -2$ 4. $k = -1$

5. a) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ b) $f'(x) = \dfrac{-6}{(2x+5)^2}$

6. a) $y' = 4\left(\frac{3x-1}{2x+5}\right)^3 \left[\frac{3(2x+5)-2(3x-1)}{(2x+5)^2}\right]$ b) $y' = 2x\left(5^{x^2}\ln 5\right) - 10x^{-3} + \frac{5}{2}x^{3/2} - \frac{2}{(2x+1)\ln 5}$

c) $y' = \frac{-\csc x \cot x\,.\,\sin x - \cos x(1+\csc x)}{\sin^2 x}$ d) $y' = -3\sin(3x)\sec(\cos(3x))\,.\,\tan(\cos(3x))$

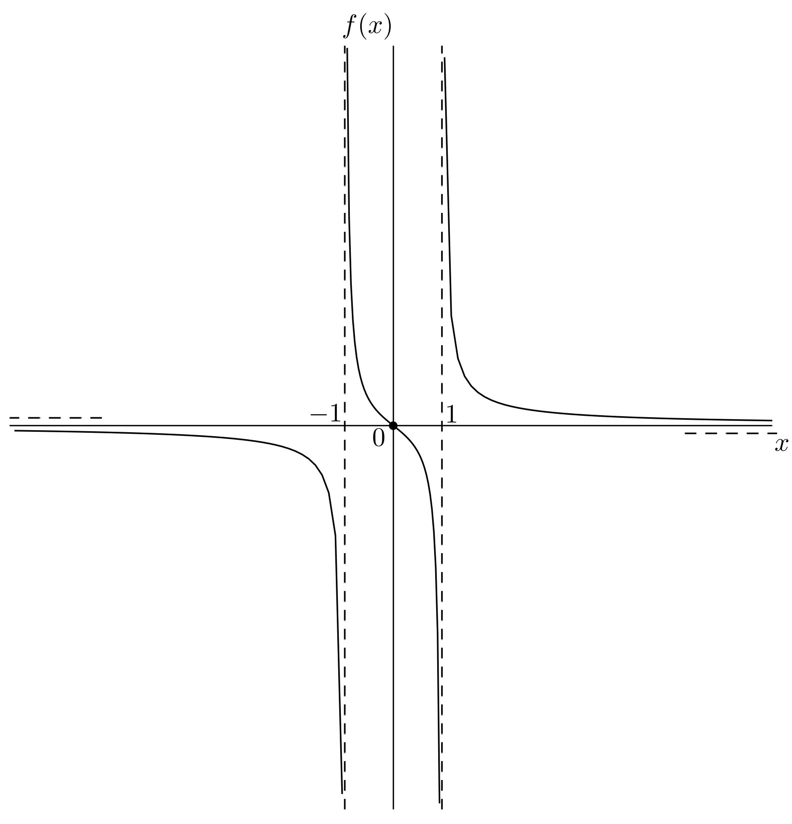
e) $y' = \sec^2 x\,.\,e^{\tan x} + e^x\,.\,\cot(x) - \csc^2(x)\,.\,e^x$ f) $y' = (x^2+1)^{2x}\left[2\ln(x^2+1) + \dfrac{2x}{x^2+1}\,.\,2x\right]$

7. $\dfrac{d^2y}{dx^2} = 2xe^{x^2}(2x^4+7x^2+3)$ 8. $y'(0,0) = 3$ 9. $y' = 2010\frac{5x^4}{x^5+3} - 3 - \frac{1}{2}\,\frac{-5x^4+6x}{6-x^5+3x^2}$

10. $y = 9x - 13$ 11. relative maximum at $(2,0)$ and relative minimum at $(4,-4)$

12. absolute maximum is 6 at $x = -1$ and absolute minimum is -26 at $x = 3$

13. x-int and y-int:(0,0)
vertical asymptotes: $x = -1$ and $x = 1$
horizontal asymptote: $y = 0$
no relative extrema; IP:$(0,0)$
Inc: never
Dec: $(-\infty,-1)\cup(-1,1)\cup(1,+\infty)$
CU: $(-1,0)\cup(1,+\infty)$
CD: $(-\infty,-1)\cup(0,1)$



14. 15 additional trees

15. a) $R'(80000) = 30$
If the production increases to 80001 units, the revenue will increase by about \$30.
b) $P = -0.0005x^2 + 70x - 5000$
c) $x = 70000$ units

16. a) $\eta = \frac{2(\sqrt{x}-18)}{\sqrt{x}} \implies \eta(100) = -1.6$
b) elastic at $x = 100$ units
c) unit elasticity at $x = 144$ units