1. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist and use $\pm\infty$ as appropriate. Show your work. (18 marks)

   a) $\lim\limits_{x\to 2}\dfrac{x^2+3x-10}{x^2-4}$

   b) $\lim\limits_{x\to 0}\dfrac{3\sin(2x)}{x}$

   c) $\lim\limits_{x\to -2^+}\dfrac{x}{x+2}$

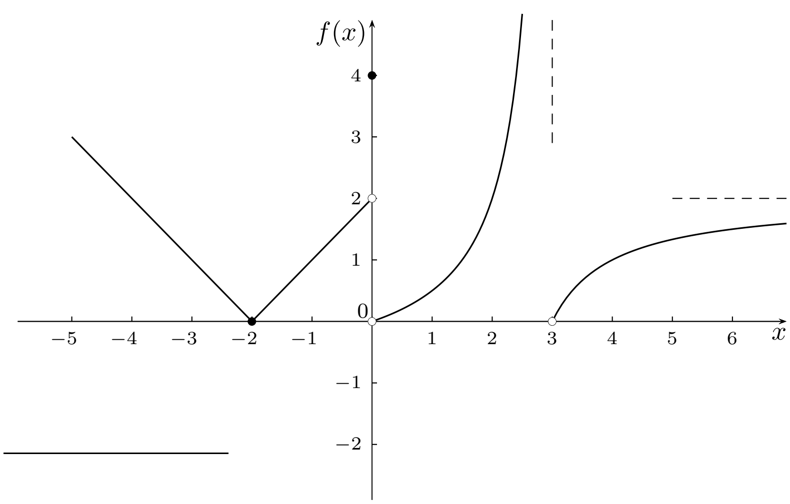
   d) $\lim\limits_{x\to 3}\dfrac{x-3}{\frac{1}{6-x}-\frac{1}{3}}$

   e) $\lim\limits_{x\to -\infty}\dfrac{(x-2)(3x^2-4)}{x(x+5)}$

   f) $\lim\limits_{x\to \infty}\dfrac{6x^2(x-3)(4-x)}{(x^2-3)(x^2-7)}$

   g) $\lim\limits_{x\to 3^-}\dfrac{5|x-3|}{x-3}$

2. Use the graph of the function $f(x)$ to find the following. Use $\pm\infty$ or DNE where appropriate. (4 marks)



   a) $\lim\limits_{x\to\infty} f(x) =$ ____

   b) $\lim\limits_{x\to 3^-} f(x) =$ ____

   c) $\lim\limits_{x\to 0} f(x) =$ ____

   d) $\lim\limits_{x\to -\infty} f(x) =$ ____

   e) $f(3) =$ ____

   f) Point(s) of discontinuity: ________________

   g) y-intercept: ____

   h) Point(s) where f is continuous but not differentiable: __________________

3. Find the point(s) of discontinuity of the function. Justify using the definition of continuity. (3 marks)

$$f(x)=\begin{cases}\dfrac{x-3}{(x-6)(x-2)} & \text{if } x<4\\[2ex] \dfrac{x+1}{5-x} & \text{if } x\geq 4\end{cases}$$

4. Find the value(s) of the constant k such that the following function $f(x)$ is continuous for all real numbers. (3 marks)

$$f(x)=\begin{cases}x^2+k^2x-4k & \text{if } x\leq 1\\[2ex] 7x+k & \text{if } x>1\end{cases}$$

5. a) State the limit definition for the derivative of a function $f(x)$. (1 mark)

b) Use the above definition to find the derivative of $f(x)=\sqrt{5-x}$ (3 marks)

6. Given $xy^3+xy=14$

a) Find y' (4 marks)

b) Find the equation of the tangent line at the point (7, 1). (4 marks)

7. Find the derivative for each of the following functions. **<u>Do not simplify your answers.</u>**

a) $y=e^2x^4-\sqrt[4]{x}+\dfrac{2}{\sqrt{x}}+e^{x^3}$ (2 marks)

b) $y=\ln\left[\dfrac{\left(x^4-6x\right)^3\left(7x-1\right)^2}{\cot^3 x}\right]$ (4 marks)

c) $y=\left(x^7-3x\right)^{\ln x}$ (4 marks)

d) $y=\dfrac{1-\tan\left(2x\right)}{1+\ln x}$ (4 marks)

e) $y=3^{x^3-1}\left(\cos\left(3x^3\right)\right)$ (4 marks)

f) $y=\left(e^{7x}+\sin^4 x\right)^5$ (4 marks)

g) $y=\log_3\left(x^2-7x\right)+\sqrt[5]{x^2}+\dfrac{1}{\left(6x^3-8\right)^2}$ (3 marks)

8. Use the Second Derivative Test to find all relative extrema of $f\left(x\right)=6x^2-6x^4$ (3 marks)

9. Given: $y = \dfrac{(x-4)(x+1)}{x^2-4}$; $y' = \dfrac{3x^2+12}{(x^2-4)^2}$; $y'' = \dfrac{-6x(x^2+12)}{(x^2-4)^3}$

(10 marks)

a) Find the x-intercept(s) and the y-intercept, if any.

b) Find the vertical asymptote(s) and horizontal asymptote(s), if any.

c) Determine all intervals where $f(x)$ is:
   (i) increasing

   (ii) decreasing

   (iii) concave up

   (iv) concave down

d) Where applicable, give the x and y coordinates of all:
   (i) relative extrema

   (ii) point(s) of inflection

e) Sketch the graph of $f(x)$. Indicate all intercepts, relative maxima, relative minima and point(s) of inflection.

10. Find the absolute extrema of $f(x) = x^3 - 12x^2 - 27x + 3$ on the interval $[0, 10]$. (3 marks)

11. A fence is to be built to enclose a rectangular area of 384 square feet. The fence along three sides is to be made of material that costs 3 dollars per foot, and the material for the fourth side costs 13 dollars per foot. Find the dimensions of the enclosure that is most economical to construct. (5 marks)

12. The manager of a large apartment complex knows from experience that 90 units will be occupied if the rent is 294 dollars per month. A market survey suggests that, on the average, one additional unit will become vacant for each 7 dollar increase in rent. What rent should the manager charge to maximize the revenue? (5 marks)

13. If the average manufacturing cost (in dollars per unit) of a product is $\overline{c} = 3x^3 + x^2 + 4x$ where x is the number of units manufactured and the selling price in dollars per unit is given by $p = 2x^3 + 25x^2 + 130x$, (5 marks)
   a) What should be the production level, x, in order to maximize the profit?
   b) What is the maximum profit?

14. The demand function for a product is given by $p = \sqrt{60 - 4x}$ for $0 \le x \le 15$. (5 marks)
   a) Find the price elasticity of demand when $x = 5$.
   b) Is the demand *elastic* or *inelastic* when $x = 5$?
   c) Find the value of x such that the demand is unit elastic.

Answers:

1) a) 7/4 b) 6 c) $-\infty$ d) 9 e) $-\infty$ f) -6 g) -5

2) a) 2 b) ∞ c) DNE d) ∞ e) DNE f) 0, 3 g) 4 h) $x=-2$

3) $x=4$ since $\lim_{x\to 4} f(x)$ DNE ; $x=2$ since $f(2)$ DNE; $x=5$ since $f(5)$ DNE

4) $k=6$, $k=-1$

5) a) $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

b) $$f'(x)=\lim_{h\to 0}\frac{\sqrt{5-x-h}-\sqrt{5-x}}{h}\cdot\frac{\sqrt{5-x-h}+\sqrt{5-x}}{\sqrt{5-x-h}+\sqrt{5-x}}$$
$$=\lim_{h\to 0}\frac{5-x-h-5+x}{h\left(\sqrt{5-x-h}+\sqrt{5-x}\right)}=\lim_{h\to 0}\frac{-h}{h\left(\sqrt{5-x-h}+\sqrt{5-x}\right)}$$
$$=\lim_{h\to 0}\frac{-1}{\left(\sqrt{5-x-h}+\sqrt{5-x}\right)}=\frac{-1}{2\sqrt{5-x}}$$

6) a) $y'=\dfrac{-y-y^3}{3xy^2+x}$ b) $y=-\dfrac{1}{14}x+\dfrac{3}{2}$

7) a) $y'=4e^2x^3-\dfrac{1}{4}x^{-3/4}-x^{-3/2}+3x^2e^{x^3}$ b) $y'=3\left(\dfrac{4x^3-6}{x^4-6x}\right)+2\left(\dfrac{7}{7x-1}\right)-3\left(\dfrac{-\csc^2 x}{\cot x}\right)$

c) $$y'=\left(x^7-3x\right)^{\ln x}\left[(\ln x)\left(\frac{7x^6-3}{x^7-3x}\right)+\left(\frac{1}{x}\right)\ln\left(x^7-3x\right)\right]$$

d) $$y'=\frac{(1+\ln x)\left(-2\sec^2 2x\right)-(1-\tan 2x)\left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

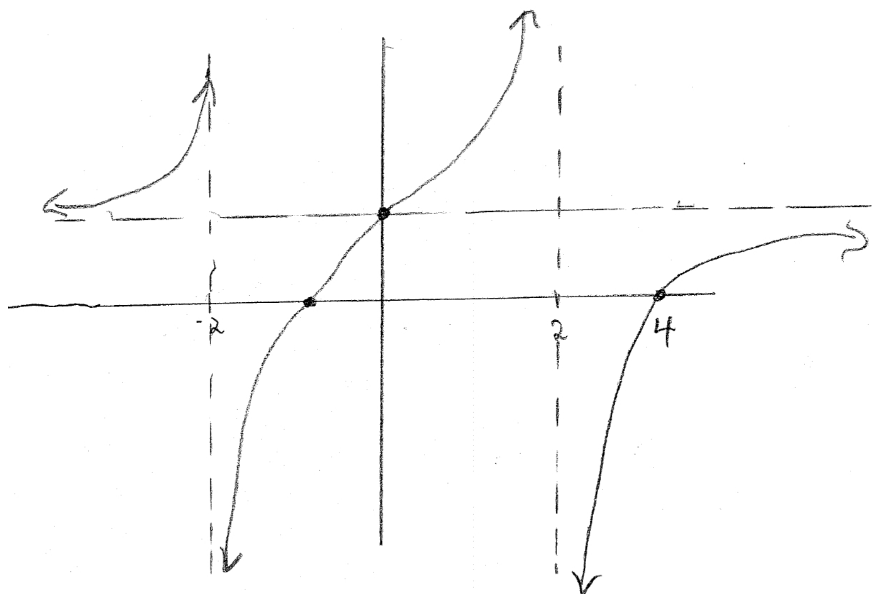
e) $$y'=3^{x^3-1}\left[-\sin\left(3x^3\right)\cdot 9x^2\right]+3^{x^3-1}\cdot\ln 3\cdot\left(3x^2\right)\cos\left(3x^3\right)$$

f) $$y'=5\left(e^{7x}+\sin^4 x\right)^4\left[7e^{7x}+4\sin^3 x(\cos x)\right]$$

g) $$y'=\frac{2x-7}{\left(x^2-7x\right)\ln 3}+\frac{2}{5}x^{-3/5}-2\left(6x^3-8\right)^{-3}\left(18x^2\right)$$

8) Relative minimum at $x = 0$; Relative maxima at $x = \pm\sqrt{\frac{1}{2}}$

9) a) x-int: (4, 0) and (−1, 0); y-int at (0, 1)
b) V.A.: $x = 2$ and $x = -2$; H.A.: $y = 1$
c) Increasing $(-\infty, -2)$ and $(-2, 2)$ and $(2, \infty)$, never decreasing
Concave Up $(-\infty, -2)$ and $(-2, 0)$; Concave Down $(0, 2)$ and $(2, \infty)$
d) No relative Maximum, No relative Minimum; IP at (0, 1)
e)



10) Absolute Max at (0, 3); Absolute Min at (10, -467)

11) 32 by 12 feet

12) \$462 (when rent is increased by \$7 24 times)

13) a) $x = 21$ b) \$83349

14) a) $\eta = -4$ b) Elastic c) $x = 10$